PH3202 - Physics for electrical engineering

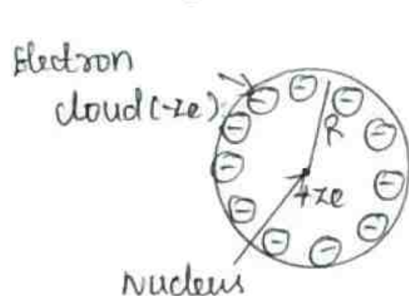
Unit-1 Dielectric Materials and insulation

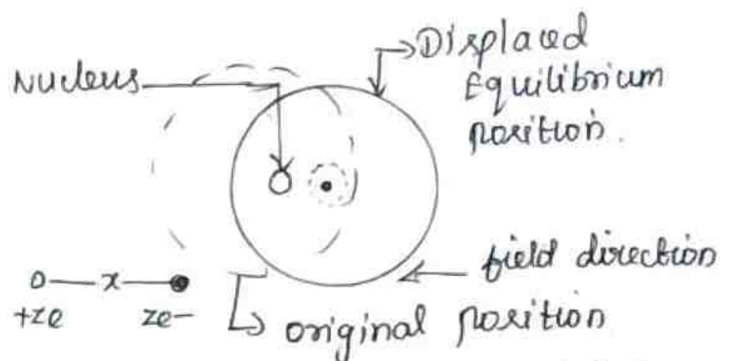
1) Polarization mechanisms in Dielectric (or) (types) of polarisation

1 Electronic polarisation:-

Electronic polarisation is due to the displacement of positively charged nucleus and negatively charged electrons of an atom in the opposite directions on the application of an electrical field.

This will result in the creation of dipole moment in the dielectric.





The electronic polarisability is given by the relation

$$\alpha_e = 4\pi\epsilon_0 R^3$$

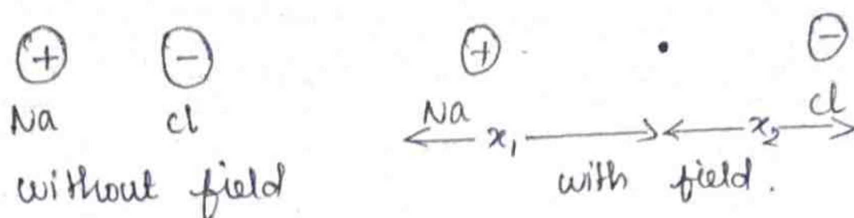
α_e is the eletronic polarisability
R is the radius of the atom.

2 Ionic polarisation:-

Ionic polarization is due to the displacement of cations positive ions (cations) and (anions) negative ions in opposite directions. eg: (NaCl)

The positive ions displace in the directions of applied electrical field through the distance x,

The negative ions displace in opposite direction through the distance x_2.



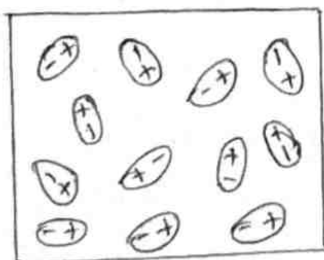
The ionic polarisability is given by.

$$\alpha_i = \frac{e^2}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right)$$

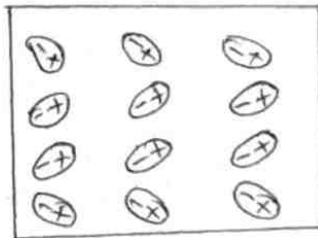
where m is the mass of the positive ion and M is the mass of the negative ion and α_i is the ionic polarisability

3. Orientation polarisation :-

The polarization arising due to the allignment of already exiting but randomly oriented dipoles in the polar substance is called the orientational or dipolar polarization.



without field



with field

The orientational polarisability is given by.

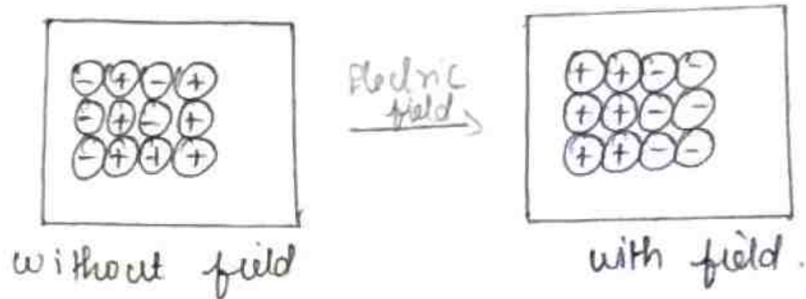
$$\alpha_0 = \frac{\mu^2}{3kT}$$

where μ is the dipole moment, k is the Boltzmann constant, T is the absolute temperature and α_0 is the orientational polarisability.

4. Interfacial (or) space charge polarisation:

Interfacial or space charge polarization is one type of dielectric polarization.

It refers to the accumulation of charges at an interface between two dielectric materials or between two regions within a material when an external field applied.



without field　　　　with field.

Example:- Space-charge polarisation occurs in semiconductors

Total polarisation:-

The total polarization of a material is the sum of the contribution from the various sources seen above.

$$P_{total} = P_e + P_i + P_o + P_s$$

Since the space-charge polarizability is very small when compared to the other types of polarizabilities, the total polarizability of a gas can be written as,

$$\alpha = \alpha_e + \alpha_i + \alpha_o$$

$$= 4\pi \varepsilon_0 R^3 + \frac{e^2}{\omega_0^2}\left(\frac{1}{M} + \frac{1}{m}\right) + \frac{\mu^2}{3kT}$$

Frequency dependence:

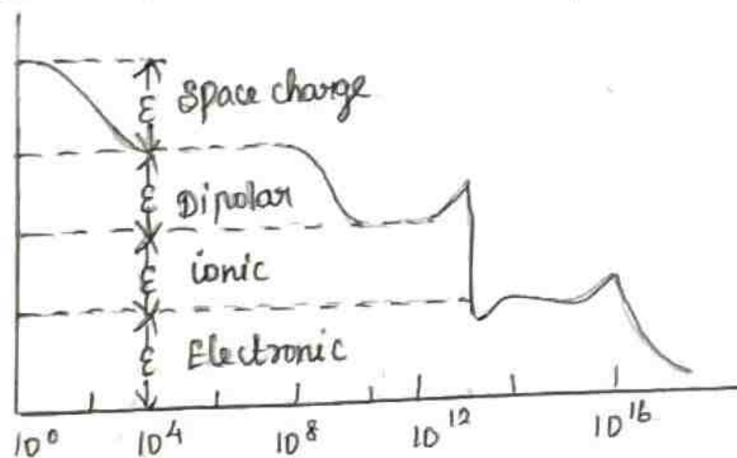
The effect of polarization depends on time and is given by.

Electronic polarization is very rapid action. this polarization exist when the frequency of the applied voltage is 10^{15} Hz.

Ionic polarization is slower than electronic polarization the frequency range with which ions can be displaced in the infrared region is equal to 10^{13} Hz.

If the frequency of the applied field is 10^{15} Hz the ions do not respond.

orientation of dipoles takes comparatively much longer time which is in the range of micro to nano seconds. As a result the orientational polarization occurs below the micro wave region.



2) <u>Local field</u>:-

when a dielectric material is placed in an external electrical field, it produces an induced dipole moment.

Now, two fields are acting at any point inside dielectrics are.

(i) macroscopic electrical field due to external electrical field.

ii) electrical field due to electric dipole moment.

There long-range coulomb field produced due to dipoles is known as internal field or local field.

Lorentz method to find internal field:

The internal field (E_{int}) at the atom site is considered to made up of the following four components E_1, E_2, E_3 & E_4.

$$E_{int} = E_1 + E_2 + E_3 + E_4 \quad \ldots (1)$$

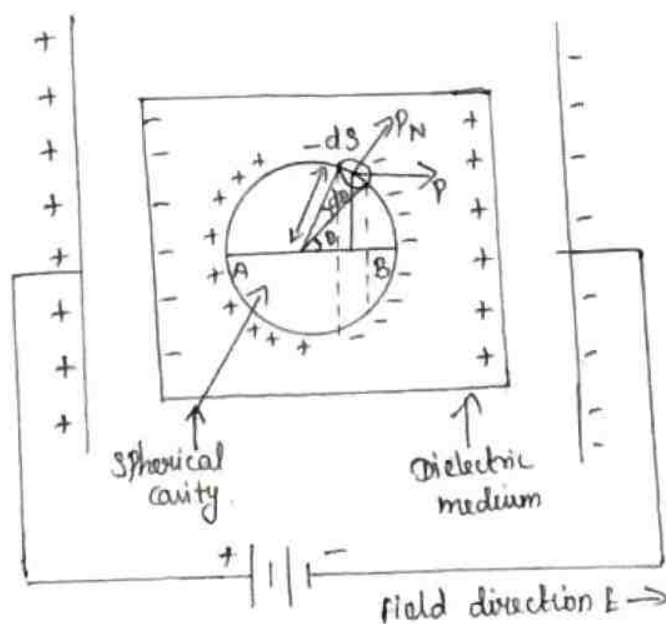
where,

E_1 - Electrical field due to charges on the plates of the capacitor.

E_2 - Electrical field due to polarised charges on the plane surface of the dielectric.

E_3 - Electrical field due to polarised charges induced on the surface of the imaginary spherical cavity.

E_4 - Electrical field due to permanent dipoles of atoms inside the spherical cavity considered.



$$E = E_1 + E_2$$

therefore, the electrical field due to permanent dipoles $E_4 = 0$

$$E_{int} = E + E_3 \quad \ldots\ldots (2).$$

$\left(\because E_{int} = E_1 + E_2 + E_3 + E_4 \right.$

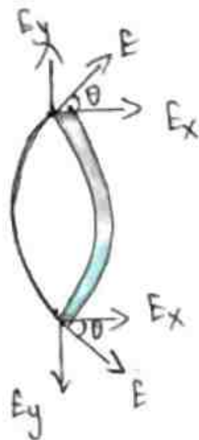
$E_1 + E_2 = E$

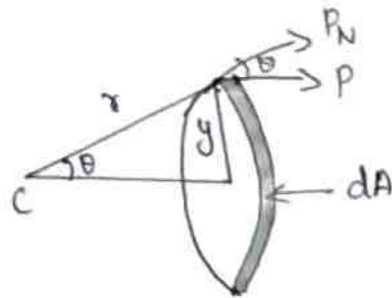
$E_4 = 0$

$\left. E_{int} = E + E_3 \right)$

Calculation of E_3 :

* Let us consider a small area "ds" on the surface of spherical cavity.
* It is confined within an angle "$d\theta$" at an angle 'θ' in the direction of electric field E.
* polarisation (p) is parallel to E. P_N is the component of polarisation perpendicular to the area "ds".
* Let 'q' is the charge on the are ds.
* polarisation is also defined as the surface charges per unit area.





$$\frac{q'}{ds}$$

$$P_N = P\cos\theta = \frac{q'}{ds}$$

charge on ds is given by

$$q' = P\cos\theta\, ds \quad \ldots \quad (3)$$

Electric field intensity at c due to charge q' is given by,

$$E = \frac{q'}{4\pi\varepsilon_0 r^2}$$

Substituting for q' from eqn (3), we have,

$$E = \frac{P\cos\theta\, ds}{4\pi\varepsilon_0 r^2} \quad \ldots \quad (4)$$

This electric field intensity is along the radius r and it can be resolved into two components

$$E_x = E\cos\theta \quad \ldots \quad (5)$$

Substituting for E from eqn, we have

$$E_x = \frac{P\cos\theta\, ds\cos\theta}{4\pi\varepsilon_0 r^2} \quad \ldots \quad (6)$$

Substituting eqn. we have

$$E_x = \frac{P\cos^2\theta\, ds}{4\pi\varepsilon_0 r^2} \quad \ldots \quad (7)$$

The component of intensity perpendicular to the field direction,

$$E_y = E\sin\theta$$

Now, consider a ring area dA which is obtained by revolving ds about AB.

Area of the ring dA = circumference x Thickness

$$dA = 2\pi y \times r\,dq \quad \left[\because \sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta\right]$$

$$dA = 2\pi r\sin\theta \times r\,d\theta$$
$$= 2\pi r^2\sin\theta\, d\theta \quad \ldots \quad (8)$$

The Electric field intensity due to the elemental ring dA is given by.

$$E = \frac{P \cos^2\theta \, dA}{4\pi \epsilon_0 r^2} \quad \dots \quad (9)$$

Substituting eqn (8) in eqn (9)

$$E = \frac{P \cos^2\theta \, 2\pi r^2 \sin\theta \, d\theta}{4\pi \epsilon_0 r^2}$$

$$E = \frac{P \cos^2\theta \sin\theta \, d\theta}{2\epsilon_0} \quad \dots \quad (10)$$

Electric field intensity due to charges present in the whole sphere is obtained by integrating eqn within the limits 0 to π.

the electric field is taken as E_3.

Substituting eqn (10) we get.

$$E_3 = \int_0^{\pi} \frac{P \cos^2\theta \sin\theta \, d\theta}{2\epsilon_0}$$

$$E_3 = \frac{P}{2\epsilon_0} \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta \qquad \because \left[\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta = \frac{2}{3} \right]$$

$$E_3 = \frac{P}{2\epsilon_0} \times \frac{2}{3}$$

$$E_3 = \frac{P}{3\epsilon_0}$$

$$E_{int} = E + E_3$$

$$E_{int} = E + \frac{P}{3\epsilon_0} \quad \dots \quad (11)$$

E_{int} is the internal field or Lorentz field.

Eqn (11) E_{int} is different from E. The local field intensity E_{int} is larger than the macroscopic field intensity E. Hence, the molecules are more effectively polarised.

3) Clausius - Mosotti equation :-

Let N be the number of molecules per unit volume and α be the molecular polarisability. Then.

Total polarisation,

$$P = N \alpha E_{int}$$

$$E_{int} = \frac{P}{N\alpha} \quad \ldots \quad (12)$$

Further, we know that,

$$D = \varepsilon E$$

$$D = \varepsilon_0 E + P \quad \ldots \quad (13)$$

$$\varepsilon E = \varepsilon_0 E + P \quad \ldots \quad (14)$$

then, $(\varepsilon - \varepsilon_0) E = P$

$$E = \frac{P}{(\varepsilon - \varepsilon_0)} \quad \ldots \quad (15)$$

Lorentz field is given by.

$$E_{int} = E + \frac{P}{3\varepsilon_0} \quad \ldots \quad (16)$$

Substituting eqn (15) in (16) we have,

$$E_{int} = \frac{P}{\varepsilon - \varepsilon_0} + \frac{P}{3\varepsilon_0}$$

$$E_{int} = P\left[\frac{3\varepsilon_0 + \varepsilon - \varepsilon_0}{3\varepsilon_0(\varepsilon - \varepsilon_0)}\right]$$

$$E_{int} = p \frac{3\varepsilon_0 + \varepsilon - \varepsilon_0}{3\varepsilon_0(\varepsilon - \varepsilon_0)} \qquad \left(\because \frac{\varepsilon + 3\varepsilon_0 - \varepsilon_0}{3\varepsilon_0(\varepsilon - \varepsilon_0)} \quad \frac{\varepsilon + 2\varepsilon_0}{3\varepsilon_0(\varepsilon - \varepsilon_0)}\right)$$

$$E_{int} = \frac{P}{3\varepsilon_0}\left(\frac{\varepsilon + 2\varepsilon_0}{\varepsilon - \varepsilon_0}\right) \quad \ldots (17)$$

equating eqns (12) & (17), we have,

$$\frac{P}{N\alpha} = \frac{P}{3\varepsilon_0}\left(\frac{\varepsilon + 2\varepsilon_0}{\varepsilon - \varepsilon_0}\right)$$

$$\frac{N\alpha}{3\varepsilon_0} = \frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} = \frac{\left(\frac{\varepsilon}{\varepsilon_0}\right) - 1}{\left(\frac{\varepsilon}{\varepsilon_0}\right) + 2}$$

$$\frac{N\alpha}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \quad \ldots (18)$$

where, ε_r is dielectric constant, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

equation (18) is Clausius - Mosotti relation which relates the dielectric constant of the material with polarisability

4) Electronic polarisation (8 mark) (or) (16 mark)

Electronic polarisation is due to the displacement of positively - charged nucleus and negatively - charged electrons of an atom in opposite directions on the application of an electrical field. This induces dipole moment in the dielectric.

Dipole moment (μ) is proportional to the electric field strength (E)

$$\mu \propto E$$

eqn (1) substituting in above eqn,

$$= -\frac{3}{4}\frac{Ze}{\pi R^3} \times \frac{4}{3}\pi x^3$$

$$= -\frac{Ze\,x^3}{R^3} \quad \ldots\ldots (4).$$

Total positive charge of atom present in the sphere of radius x,

$$Q_p = +Ze$$

Substituting eqn (4) in (3),

$$F_c = -\frac{Ze\left(-\frac{Zex^3}{R^3}\right)}{4\pi\varepsilon_0 x^2}$$

$$F_c = \frac{-Z^2e^2x}{4\pi\varepsilon_0 R^3} \quad \ldots (5).$$

At equilibrium, Coulomb force and Lorentz force must be equal and opposite,

$$F_L = -F_c$$

substituting for F_L and F_c from equation (2) and (5),

$$ZeE = -\left(-\frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}\right)$$

$$E = \frac{Zex}{4\pi\varepsilon_0 R^3}$$

$$x = \frac{4\pi\varepsilon_0 R^3 E}{Ze} \quad \ldots\ldots (6).$$

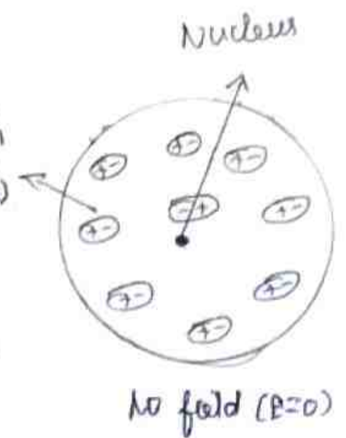
From the definition of dipole moment, induced dipole moment (μ_{ind}) is given by

$$\mu = \alpha_e E$$

where α_e is proportionality constant and it is known as electronic polarisability.

i) without electric field:

where z is the atomic number



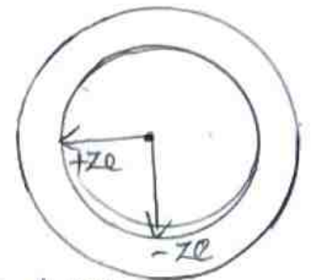
Negative charge density of atom is given by

$$\rho = \frac{\text{Total negative charge}}{\text{volume of the atom}} = \frac{-Ze}{\frac{4}{3}\pi R^3}$$

$$\rho = -\frac{3}{4}\frac{Ze}{\pi R^3} \quad \ldots \quad (1)$$

ii) with electric field:-

when the atom of the dielectric is placed in an electrical field of strength E.



The electron cloud and the nucleus are separated by a distance 'x'. It results in formation of electrical dipole in the atom.

Lorentz force between nucleus and electron $\Big\} F_L$ = charge × electrical field

$$= ZeE \quad \ldots \quad (2)$$

Coulomb attractive force (F_C) between nucleus and electron cloud being separated at a distance x.

$$F_C = \frac{Q_p\, Q_e}{4\pi\varepsilon_0 x^2} \quad \ldots \quad (3)$$

Q_e = Charge density (ρ) × Volume of the sphere of radius x

$$= \rho \times \frac{4}{3}\pi x^3$$

$$\mu_{ind} = \text{Magnitude of charge} \times \text{Displacement}$$

$$\mu_{ind} = Zex \quad \ldots (7)$$

substituting the value of x from eqn (6) in (7),

$$\mu_{ind} = Ze \cdot \frac{4\pi\varepsilon_0 R^3 E}{Ze}$$

$$\mu_{ind} = 4\pi\varepsilon_0 R^3 E \quad \ldots (8)$$

The induced dipole moment in terms of polarisability is given by,

$$\mu_{ind} = \alpha_e E \quad \ldots (9)$$

α_e is called as electronic polarisability.

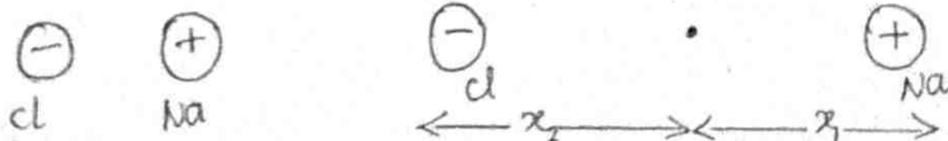
on comparing eqns (8) & (9),

$$\frac{\mu_{ind}}{\mu_{ind}} = \frac{4\pi\varepsilon_0 R^3 E}{\alpha_e E} = \frac{4\pi\varepsilon_0 R^3}{\alpha_e}$$

$$\alpha_e = 4\pi\varepsilon_0 R^3 \quad \ldots (10)$$

5) Ionic polarisation :-

Ionic polarisation is due to the displacement of cations and anions in opposite directions. This occurs in ionic dielectrics by influence of external electrical field.



without electric field   with electric field.

The net distance between two ions

$$x = x_1 + x_2 \quad \ldots (1)$$

(18)

The restoring force produced is proportional to the displacement.

For positive ion:-

Restoring force $F \propto x_1$

Restoring force acting on the ~~neg~~ positive ion } $F = \beta_1 x_1$ . . . (2)

For negative ion:-

Restoring force $F \propto x_2$

Restoring force acting on the negative ion } $F = \beta_2 x_2$ . . . (3)

where β_1 and β_2 are restoring force constants

If m is the mass of positive ion, M is the mass of the negative ion and ω_0 is the angular frequency.

$$\beta_1 = m\omega_0^2 \quad \dots (4)$$

$$\beta_2 = M\omega_0^2 \quad \dots (5)$$

Substuting eqn (4) in eqn (2)

$$F = m\omega_0^2 x_1 \quad \dots (6)$$

From force $F = eE$ . . . (7)

eqn (7) in eqn (6).

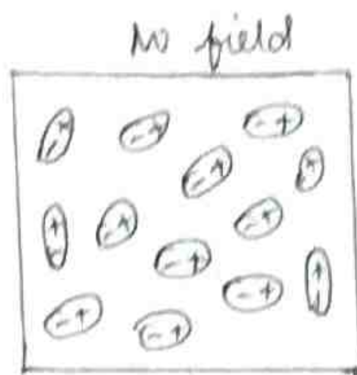
$$eE = m\omega_0^2 x_1$$

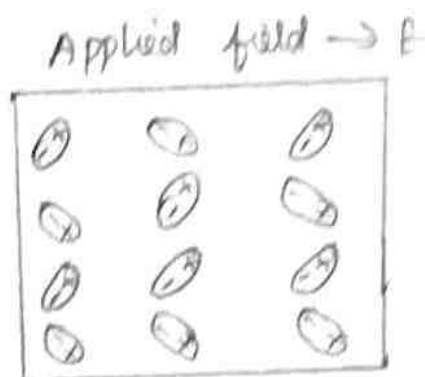
$$x_1 = \frac{eE}{m\omega_0^2} \quad \dots (8)$$

Similarly for the negative ion, we can write,

$$x_2 = \frac{eE}{M\omega_0^2} \quad \dots (9)$$

(14)





The contribution to polarisation due to orientation of molecular dipoles is called orientational polarisation.

The Langevin's theory of paramagnetism, net intensity of magnetisation = $\frac{N\mu^2 B}{3kT}$

Since the same principle can be applied to the application of electric field in dielectrics,

Orientational polarisation $P_o = \frac{N\mu^2 E}{3kT}$ . . . (1)

But, orientational polarisation is proportional to applied field ($\vec{E}$) and it is given by,

$$P_o = N\alpha_o \vec{E} \quad \ldots (2)$$

comparing eqns (1) & (2)

$$\alpha_o = \frac{\mu^2}{3kT} \qquad \left[\frac{P_o}{P_o} = \frac{\frac{N\mu_o^2 E}{3kT}}{N\alpha_o \vec{E}} \right]$$

where α_o is orientational polarisability $\alpha_o = \mu^2/3kT$

Example:-

Orientational polarisation occurs only in polar dielectrics such as H_2O, HCl, CO etc.

Adding equations (8) and (9)

$$x = x_1 + x_2 = \left(\frac{eE}{m\omega_0^2} + \frac{eE}{M\omega_0^2}\right)$$

$$= \frac{eE}{\omega_0^2}\left[\frac{1}{m} + \frac{1}{M}\right] \quad \ldots (10)$$

$$\mu = e \times x \quad \ldots (11)$$

substituting eqn (10) in eqn (11)

$$\mu = e \times \frac{eE}{\omega_0^2}\left[\frac{1}{m} + \frac{1}{M}\right] \ldots$$

$$\mu = \frac{e^2 E}{\omega_0^2}\left[\frac{1}{m} + \frac{1}{M}\right] \ldots (12)$$

$$\mu \propto E$$

$$\mu = \alpha_i E \quad \ldots (13)$$

α_i is ionic polarisability of dielectric material.

comparing equations (12) & (13).

$$\frac{\mu}{\mu} = \frac{\frac{e^2 E}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right)}{\alpha_i E}$$

$$\alpha_i = \frac{e^2}{\omega_0^2}\left(\frac{1}{m} + \frac{1}{M}\right) \ldots (14)$$

6) Orientational polarisation:-

Orientational polarisation takes place only in polar dielectrics. polar dielectrics have molecules with permanent dipole moments even in the absence of an electrical field.

iii) Polarisability (α)

It is found that the average dipole moment ($\bar{\mu}$) of a system is proportional to the applied electric field (E)

$$\bar{\mu} \propto E \quad (\text{or}) \quad \bar{\mu} = \alpha E$$

where α is proportionality constant and it is known as the polarisability

$$\boxed{\alpha = \frac{\bar{\mu}}{E}}$$

polarisability is defined as the ratio of average dipole moment to the electric field applied. Its unit is farad m^2 or Fm^2.

iv) Polarisation vector ($\vec{P}$)

It is defined as the average dipole moment per unit volume of a dielectric.

If N is the number of atoms per unit volume and $\bar{\mu}$ is average dipole moment per atom, then

polarisation vector $\boxed{\vec{P} = N\bar{\mu}}$ its unit is coulomb/m^2

Electric displacement vector $\vec{D}$

It is independent of the nature of medium. This electric vector is known as electric displacement vector $\vec{D}$

The electric displacement vector is used for analysing electrostatic fields in the presence of dielectrics.

It is given by $\boxed{\vec{D} = \frac{q}{4\pi r^2}}$ . . . (1)

we know that electric field intensity.

$$\vec{E} = \frac{q}{4\pi \varepsilon r^2} \quad \ldots \quad (2)$$

(17)

7) Dielectric constant (ε_r)

(i) Dielectric characteristics of a material is determined by its dielectric constant. It is a measure of polarisation in the dielectrics.

Definition:

It is the ratio of absolute permittivity of the medium (ε) and permittivity of free space (ε_0)

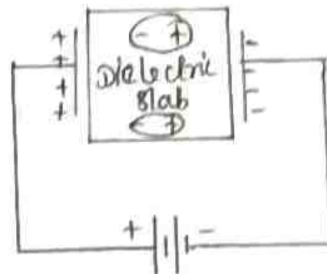
$$\text{Dielectric constant } \varepsilon_r = \frac{\text{Absolute permittivity } (\varepsilon)}{\text{Permittivity of free space } (\varepsilon_0)}$$

$$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

The value of $\varepsilon_r = 1$ for air or vacuum. Dielectric constant is also called as relative permittivity.

(ii) Polarisation:

The electrical behaviour of a dielectric material can be changed by the application of an external electrical field.



The positive charges are pushed in the same direction as that of the electrical field while negative charges are pushed in the opposite direction.

Definition:

The process of producing electrical dipoles inside the dielectric by the application of an external electrical field is called polarisation in dielectrics.

(18)

(or) $\varepsilon E = \frac{q}{4\pi r^2}$

From eqn (1) and (2),

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \quad \dots (3) \qquad (\because \varepsilon = \varepsilon_0 \varepsilon_r)$$

$$\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} \qquad (\because \varepsilon_r = 1 + \chi_e)$$

where χ_e is electrical susceptibility.

$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad \dots (4)$$

eqn (3) and (4).

$$\varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \varepsilon_0 \varepsilon_r \vec{E} - \varepsilon_0 \vec{E}$$

$$\vec{P} = \vec{E} \varepsilon_0 (\varepsilon_r - 1)$$

$$\boxed{\frac{\vec{P}}{\vec{E}} = \varepsilon_0 (\varepsilon_r - 1)} \quad \dots (5)$$

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8) Dielectric Loss:-

* When a dielectric is applied with AC voltages, the electrical energy is absorbed by with dielectric and certain quantity of electrical energy is dissipated in the form of heat energy.

* This dissipation of electrical energy is known as dielectric loss.

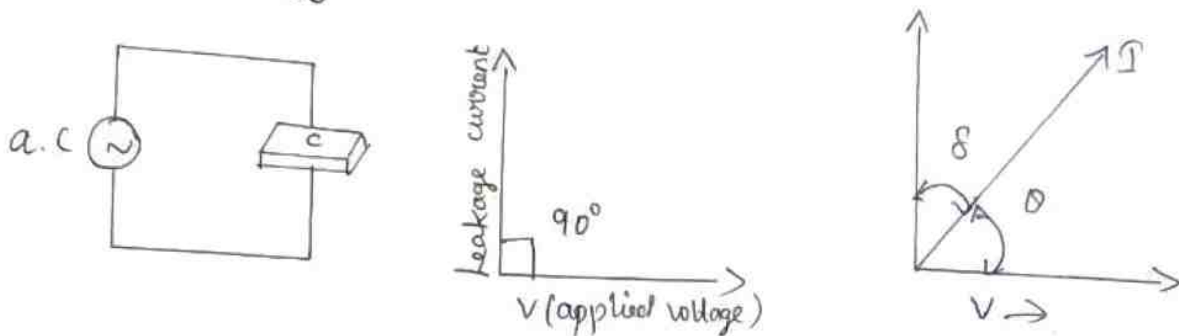
* Dielectric loss can occur in both direct and alternating voltages.

(20)

It is less in direct voltage than that of alternating voltage

Expression for dielectric loss :-

When an AC voltage is applied to a perfect dielectric like vacuum or purified gas. it does not absorb electrical energy and there is no loss of electrical energy.



we know that power loss.

$$P_L = VI \cos\theta$$

when $\theta = 90°$, $P_L = VI \cos 90°$

$$P_L = 0 \qquad (\because \cos 90° = 0)$$

This means that there is no power loss in the perfect dielectric.

* The phase angle (θ) is always less than 90°. as
* The current leads the voltage by (90 − δ).
* δ is called dielectric loss angle.
* The power loss in a dielectric having a capacitance C for applied voltage V of frequency f is given by.

$$P_L = VI \cos\theta$$

$$\theta = 90 - \delta$$

(19)

$$P_L = VI \cos(90° - \delta) \quad \ldots (*)$$

$$P_L = VI \sin\delta \quad \ldots (1) \quad (\because \cos(90° - \delta) = \sin\delta)$$

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

Similarly, for capacitor if the capacitive reactance is X_c,

$$I = \frac{V}{X_c} \quad \ldots (2) \quad (\because R = X_c)$$

X_c depends on frequency of applied a.c voltages and capacitance. It is given by,

$$X_c = \frac{1}{2\pi f c} \quad \ldots (3)$$

Substituting eqn (3) in (2),

$$I = \frac{V}{\frac{1}{2\pi f c}} = V\, 2\pi f c \quad \ldots (4)$$

Substituting eqn (4) in (1),

$$P_L = VV\, 2\pi f c \sin\delta$$

$$P_L = V^2\, 2\pi f c \sin\delta \quad \ldots (5)$$

In most of the dielectrics, the angle δ is very small.

$$\therefore \sin\delta = \tan\delta$$

Dielectric power loss, $P_L = 2\pi f c\, V^2 \tan\delta \quad \ldots (6)$

$\tan\delta$ is called the power factor of the dielectric.

9) Dielectric Breakdown:-

The dielectric loses its insulating property and becomes a conductor. This phenomenon is known as dielectric breakdown.

Dielectric strength:-

The electric field strength at which dielectric breakdown occurs is known as dielectric strength.

It is the breakdown voltage per unit thickness of the material

$$\text{Dielectric Strength} = \frac{\text{Dielectric breakdown voltage}}{\text{Thickness of dielectric}}$$

In practical applications, the failure or breakdown of a dielectric material is of great concern to engineers.

There are different mechanisms by which dielectric breakdown takes place.

Types of dielectric breakdown:-

Some important types of dielectric breakdown are,

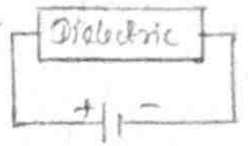
* Intrinsic breakdown and avalanche breakdown
* Thermal breakdown
* chemical and electrochemical breakdown
* Discharge breakdown
* Defect breakdown.

Intrinsic breakdown:-

In dielectric, the charge displacement increases with increasing electrical field strength.

(22)

when the applied electric field is large, some of the electrons in the valence band cross over to the conduction band across the large forbidden energy gap.



As a result, a large current flows through the dielectric and breakdown occurs. This type of breakdown is called intrinsic breakdown.

Avalanche breakdown:-

This process continues as more and more valence electrons jump to the conduction band by breaking the covalent bond.

As a result a large a current flows through the dielectric and breakdown occurs. This type of breakdown is called avalanche breakdown.

characteristics:-

* Intrinsic and avalanche breakdown require large electrical field.
* They occur even at low temperatures.
* They can occur in thin samples.

ii) Thermal breakdown:-

When an electric field is applied to a dielectric material, some amount of heat is produced. This heat must be dissipated from the material.

During this process, a large amount of current flows through the material and causes the dielectric to breakdown. This type of breakdown is known as thermal breakdown.

characteristics :-

* This type of breakdown occurs only at high temperature.

* The breakdown time is of the order of few milliseconds.

* It requires moderate electrical fields.

iii) chemical and electrochemical breakdown :-

This leads to leakage current and energy loss in the material and finally dielectric breakdown occurs.

This type of breakdown is known as chemical and electrochemical breakdown.

characteristics :-

* It occurs even at low temperatures

* It depends on the concentration of ions and magnitude of leakage current.

iv) Discharge breakdown :-

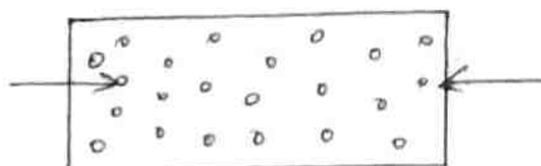
Discharge breakdown occurs when a dielectric contains occluded gas bubbles.

The gaseous ions bombard the solid dielectric. This causes electrical determination and leads to dielectric breakdown. This is known as discharge breakdown.

characteristics :-

* It occurs at low voltages.

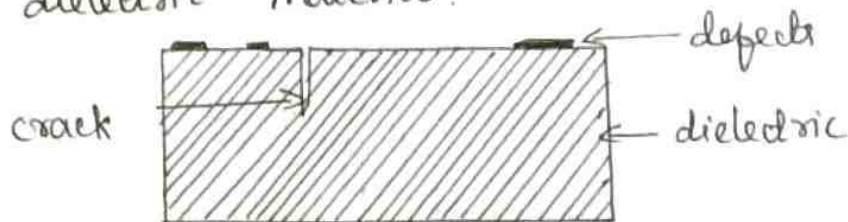
* It occurs in the dielectric material where there are a large number of occluded gas bubbles.



(V) Defect breakdown:-

The surface of the dielectric material may have defects such as cracks, porosity and blow holes.

Impurities like dust or moisture may present at these discontinuities. This will lead to a breakdown in a dielectric material.



Remedies for dielectric breakdown:-

To avoid breakdown, the insulating material should have the following properties.

* It should have high resistivity to reduce leakage current.
* It should have high dielectric strength to withstand high voltage.
* It should have small dielectric loss
* It should have sufficient mechanical ~~loss~~ strength
* It should be resistant to oils, liquids, gas fumes, acids and alkalies.
* It should have small thermal expansion to prevent mechanical damage.
* It should be fire - proof.

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10) Capacitor Materials - Typical capacitor construction

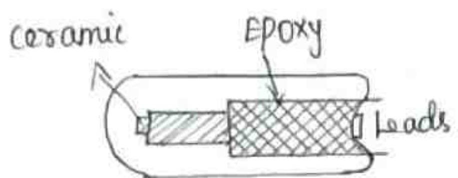
* Dielectric materials are used to manufacture capacitors of different ranges.

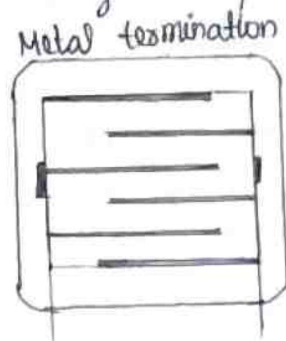
* The selection of right dielectric materials for capacitor applications is based on various parameters value of capacitance, frequency, tolerance. Size and operating voltage

1. Single and multilayer dielectric capacitors:

* The diagramatic representation of a single layer capacitor. It consists of a thin ceramic disk or plate placed in-between the metal electrodes.

* The capacitor with stacking of ceramic plates in different layers is known as multilayer capacitor.





2. polymeric film capacitors:-

* Polymer thin film capacitors are as mid-frequency capacitors.

* The two metal coated polymer films are placed in parallel

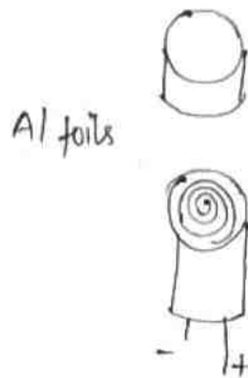
* In order to get the capacitor, the two metal sheets are rolled together like swiss roll.

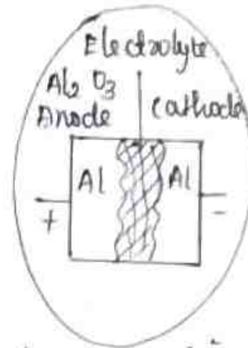
3. Electrolytic capacitors:-

* A large values capacitors are achieved using electrolytic capacitors.

* These capacitors are of different types namely, aluminium electrolytic and solid electrolyte tantalum capacitors.

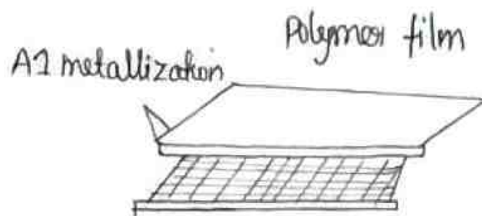
* A thin layer of Al_2O_3 as a dielectric medium is grown on rough surface of one of the foils.



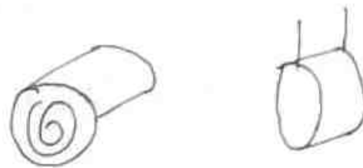


Cross sectional area

The Al_2O_3 is grown on the Al foil electrolytically and hence, the capacitor is known as electrolytic capacitor.

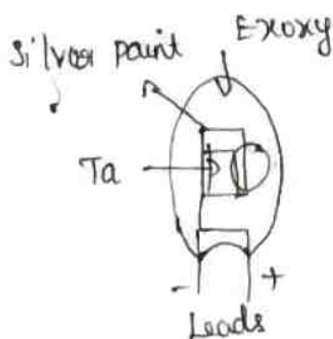


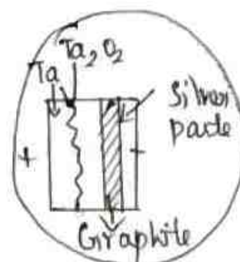
Aluminium coating on film



Capacitor

A thick solid electrolyte like MnO_2 is coated above Ta_2O_2. The process is completed by coated graphite and silver paste.





11) Ferroelectricity :-

* There are certain materials which exhibit electric polarisation even in the absence of the applied electrical field.

* They are known as ferroelectric materials. There are permanent dipole moment in each atom or molecule.

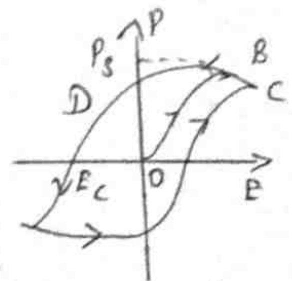
* Materials which exhibit electric polarization even in the absence of the applied electric field are known as ferroelectric materials.

Examples :-

* Barium titanate ($BaTiO_3$)
* Pottassium dihydrogen phosphate (KH_2PO_4)
* Ammonium dihydrogen phosphate ($NH_4H_2PO_4$)
* Lithium Niobate ($LiNbO_3$)
* Rochelle salt ($NaKC_4H_4O_6 \cdot 4H_2O$)

Dielectric hysteresis :-

Lagging of polarisation behind the applied electric field is called dielectric hysteresis. ferroelectricity is a result of dielectric hysteresis.



$$\chi_e = \frac{C}{T-\theta} \quad \text{---} (1)$$

properties of ferroelectric materials

* These materials have permanent dipole moment in each atom or molecule.

* Ferroelectric materials can be easily polarised even by very weak electrical field.

* They exhibit domain structure as in the case of ferromagnetic materials.

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Application of ferroelectric materials:

* Ferroelectric materials are used to make pressure transducers, ultrasonic transducers, microphones and gas fillers.
* In optical communication, the ferroelectric crystals are used for optical modulation.
* They can be used as memory devices in computers.
* They are used in electrotransducers such as microphones.

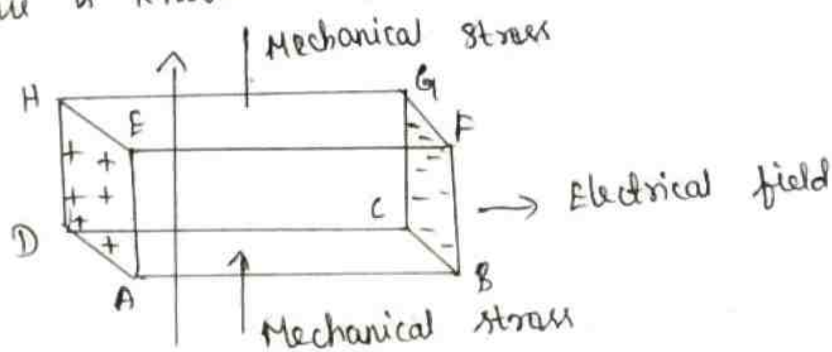
piezo - Electricity:

These materials have the property of becoming electrically polarized when mechanical stress is applied. This property, known as piezoelectric effect and has an inverse.

Explanation:

A rectangular slab is cut from a quartz crystal such that two opposite faces ABCD and EFGH are perpendicular to optic axis.

When the mechanical stress is applied to two opposite faces ABCD and EFGH, Then opposite electrical charges are developed on the other pair of opposite faces AEHD and BFGC. This is known as piezo- electric effect



There is expansion or contraction of the crystal, this is known as inverse piezo - electric effect

12) Crystal Oscillator

The frequency of LC and RC oscillators may change with time, temperature changes etc.

Some naturally available crystals have the property that their resonant frequencies are constant.

The values of the L, C, R and C' depends upon the crystal cut, its and the nature of the vibrations involved.

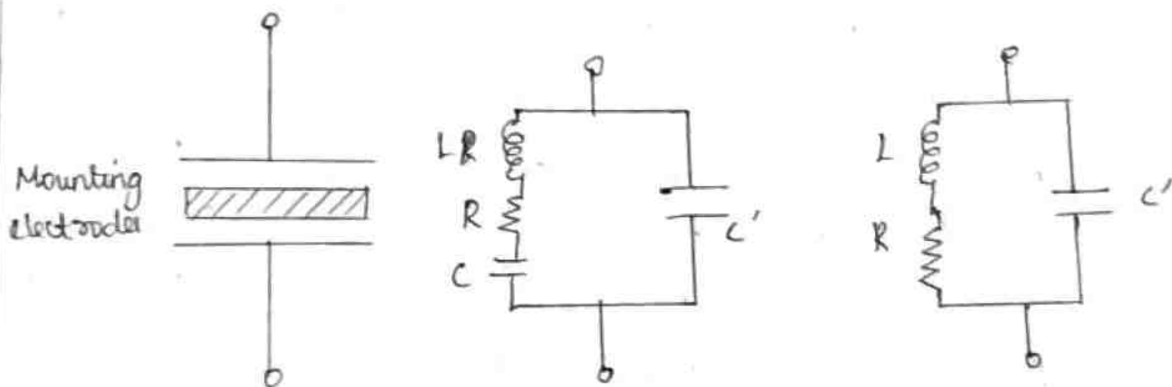
The series resonance frequency is given by.

$$f_s = \frac{\omega_s}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad \ldots (1)$$

f_s is called the series resonant frequency.

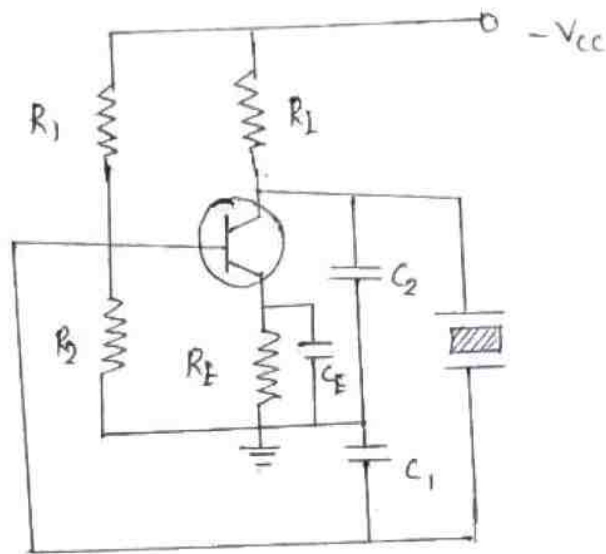
$$f_p = \frac{\omega_p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{L}\left(\frac{C'+C}{CC'}\right)} \quad \ldots (2)$$

$$= \frac{1}{2\pi}\sqrt{\frac{C'+C}{LCC'}}$$



$$f_p = \frac{1}{2\pi\sqrt{LC}} \quad \ldots (3)$$

In general crystal can be used in the frequency range from about 15kHz to 10 MHz.



Advantages of crystal oscillators:-

* By changing the crystal different frequencies can be achieved.
* The frequency drift can be made less than 1 part in 10^6
* crystal oscillator does not need a separat tuned circuit

Disadvantages:-

* If excessive power is applied, the oscillator waveform will be distorted.
* A crystal oscillator is used in low power circuits.

Application and uses:

* They are used to generate a chromatic sub-carrier or color sub-carrier in TV receivers.
* They got wide applications in microprocessors, microcontrollers, computer-clock pulse generator in radio and communication equipment.

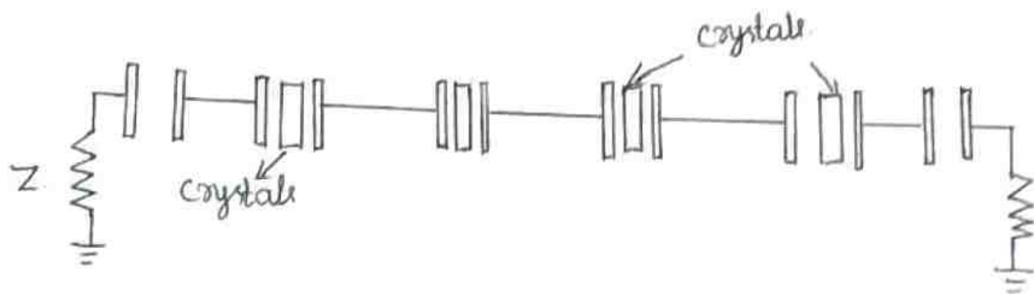
Crystal filter:-

The crystal filters are used in radio communicati telecommunications, signal generation, GPS devices and many electronic devices.

Working:-

Crystal filters are electronic filters that use monolithic crystals. A monolithic crystal filter uses one large crystal rather than an array of several discrete crystals.

Only the signals within the band-pass of the filter are allowed across the crystal. This crystal band-pass filter operates efficiently at a frequency below 30MHz



Applications:-

* crystal filters are used in the intermediate frequency stages of high-quality radio receivers
* They are very stable mechanically
* The most common use of crystal filters are at frequencies of 9MHz or 10.7 MHz to provide selectivity in communications receivers.

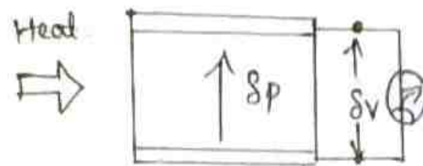
Pyroelectricity:-

The pyroelectric coefficient λ is defined as the change in polarization per unit temperature change of the specimen.

$$\lambda = \frac{dP}{dT}$$

The change in polarization results in change in external field hence charge on the surface

The Ti^{4+} ion becomes shifted, which results in a change in the polarization.



Eg:- ~~Barium~~ Barium Titanate, Lithium Niobate.

13) Pyroelectric crystals and their Applications:

The pyroelectric materials such as $BaTiO_3$, $LiNbO_3$ etc. are used to make very Good infra-red detectors which can operate at room temperature.

Ferroelectric energy convertor:

It is a device which converts heat energy into electrical energy based on pyro-electric effect.

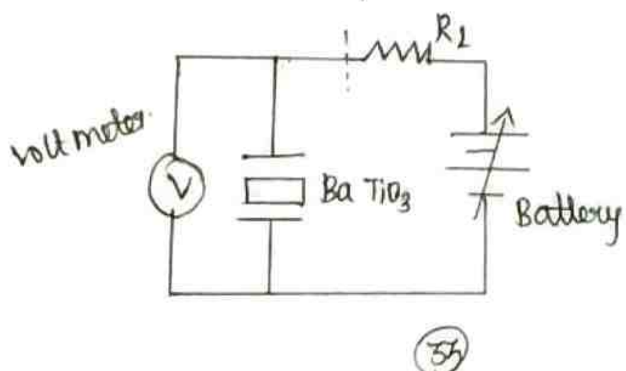
principle:-

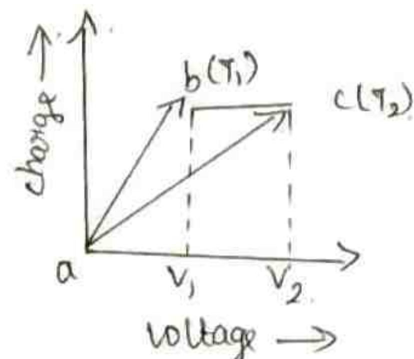
The ferroelectric crystals exhibit the phenomenon of pyroelectric effect, (i.e) change in the temperature of the crystal produces a change in its polarisation.

using this effect, one can convert heat energy into useful electrical energy.

Description:-

Ferroelectric crystal $BaTiO_3$ is placed in between two parallel plate capacitors C_1 and C_2.





Initial energy $(E_1) = \frac{1}{2} C_1 V_1^2$ = area of the triangle abc.

$$E_2 = \frac{1}{2} C_2 V_2^2$$

Net increase in energy,

$$\Delta E = E_2 - E_1$$

$$= \frac{1}{2} (C_2 V_2^2 - C_1 V_1^2)$$

Unit - 1

Dielectric Materials and insulation

problems

1. A parallel plate capacitor consists of two plates each of area $5 \times 10^{-4}\ m^2$. They are separated by a distance $1.5 \times 10^{-3} m$ and filled with a dielectric of relative permittivity 6. calculate the charge on the capacitor if it is connected to a 100 volt DC supply.

Solution :-

$$Q = CV$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$Q = \frac{\varepsilon_0 \varepsilon_r A V}{d}$$

Substituting the given values,

$$Q = \frac{8.85 \times 10^{-12} \times 6 \times 5 \times 10^{-4} \times 100}{1.5 \times 10^{-3}}$$

$$Q = 1.77 \times 10^{-9} C$$

charge on the capacitor $= 1.77 \times 10^{-9}$ coulomb.

2. If a NaCl crystal is subjected to an electrical field of 1000 V/m and the resulting polarisation is $4.3 \times 10^{-8}\ c/m^2$, calculate the relative permittivity of NaCl.

Solution :-

$$P = \varepsilon_0 (\varepsilon_r - 1) E$$

$$(\varepsilon_r - 1) = \frac{P}{\varepsilon_0 E}$$

$$\varepsilon_r = 1 + \frac{P}{\varepsilon_0 E}$$

Substituting the given values,

$$\varepsilon_r = 1 + \frac{4.3 \times 10^{-8}}{8.85 \times 10^{-12} \times 1000}$$

$$\varepsilon_r = 1 + 4.86$$

$$\varepsilon_r = 5.86$$

3. Calculate the electronic polarisability of argon atom given $\varepsilon_r = 1.0024$ at NTP and $N = 2.7 \times 10^{25}$ atoms/m^3.

Solution:-

$$P = \varepsilon_0(\varepsilon_r - 1)E$$

$$P = N\alpha_e E$$

$$N\alpha_e E = \varepsilon_0(\varepsilon_r - 1)E$$

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$$

substituting the given values,

$$\alpha_e = \frac{(8.85 \times 10^{-12})(1.0024 - 1)}{2.7 \times 10^{25}}$$

$$\alpha_e = 7.9 \times 10^{-40} \, Fm^2$$

4. The dielectric constant of he gas at NTP is 1.0000684. Calculate the electronic polarisability of the atoms if the gas contains 2.7×10^{25} atoms per m^3.

Solution:-

Electronic polarisability is given by,

$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$$

substituting the given values.

$$\alpha_e = \frac{8.85 \times 10^{-12}(1.0000684 - 1)}{2.7 \times 10^{25}} = 2.242 \times 10^{-41} \, Fm^2$$

unit - ii

* Electrical and magnetic properties of Materials

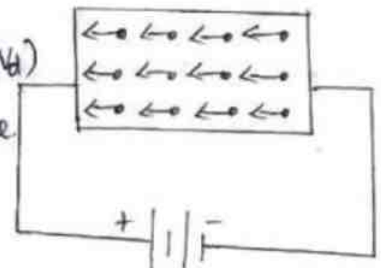
1. Expression for Electrical conductivity :-

when an electrical field (E) is applied to an electron of charge 'e' of a metallic rod.

The electron moves in opposite direction to the applied field with a velocity V_d. This velocity is known as drift velocity.

Derivation of electrical conductivity :-

The electrons are accelerated and suffer elastic collision with lattice and finally attain a constant drift velocity V_d.



$$\text{Acceleration of electron}(a) = \frac{\text{Drift velocity}(V_d)}{\text{Relaxation time}(\tau)}$$

$$a = \frac{V_d}{\tau} \quad \ldots (1)$$

Therefore, magnitude of force 'F' experience by the electron under the electric field E is given by.

$$F = eE \quad \ldots (2)$$

According to Newton's second law of motion, the force (F) acting on the electron.

$$F = ma \quad \ldots (3)$$

Substitute eq (1) in (3).

$$F = \frac{m V_d}{\tau} \quad \ldots (4)$$

①

from eqn (2) and (4).

$$\frac{mV_d}{\tau} = eE \quad \ldots\ldots (4)$$

$$V_d = \left(\frac{e\tau}{m}\right) E \quad \ldots\ldots (5)$$

The current density 'J' is related to the drift velocity a as,

$$J = neV_d \quad \ldots\ldots (6)$$

Substitute eqn (5) in eqn (6).

$$J = ne\left(\frac{e\tau}{m}\right) E \quad \ldots\ldots (7)$$

$$J = \left(\frac{ne^2\tau}{m}\right) E$$

$$\frac{J}{E} = \left(\frac{ne^2\tau}{m}\right) \quad \ldots\ldots (8)$$

from microscopic form of ohm's law, the current density 'J' is expressed as $J = \sigma E$

$$\frac{J}{E} = \sigma \quad \ldots\ldots (9)$$

From eqn (9) and (8).

$$\sigma = \left(\frac{ne^2\tau}{m}\right) \quad \ldots\ldots (10)$$

Then eqn (10) is Electrical conductivity.

From eq (10), we know that with increase of electron concentration 'n', the conductivity 'σ' increases. As 'm' increases, the motion of electron becomes slow and hence the conductivity 'σ' decreases.

②

2. Expression for Thermal conductivity (k) of an Electron:

Definition:-

Thermal conductivity (k) of a metal is defined as the amount of heat (Q) conducted per unit area (A) per unit time (t) maintaining at unit temperature gradient between two ends.

Thermal conductivity.

$$k = \frac{Q}{A(T_1 - T_2)t} 2\lambda \ W m^{-1} k^{-1}$$

where k is the thermal conductivity of the material

Q is the amount of heat energy.

$\frac{dT}{dx} = \frac{(T_1 - T_2)}{2\lambda}$ is the temperature gradient.

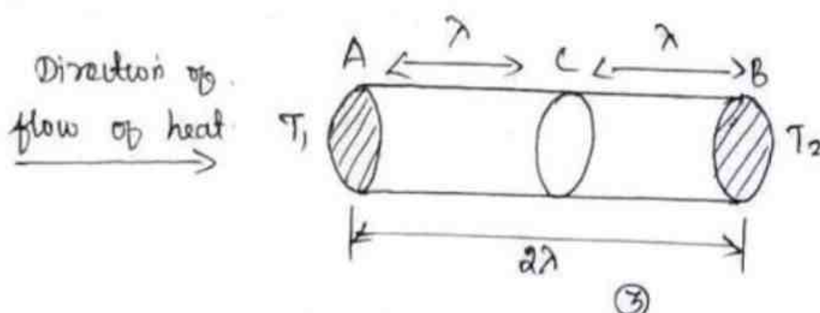
Time t = 1 sec

A = 1 unit area.

then the thermal conductivity, $k = \frac{Q}{\left(\frac{dT}{dx}\right)}$

Derivation of Thermal conductivity Metal:

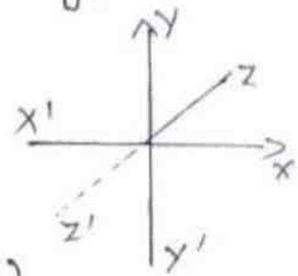
Let A and B be the two cross sections of temperatures T_1 and T_2 separated by a distance of mean free path (2λ). Heat flows from hot end 'A' to the cold end 'B'.



③

The amount of heat conducted (Q) by the rod from the end A to B of length 2λ is given by,

$$k = \frac{Q}{A(T_1 - T_2)t} 2\lambda$$



At time $t = 1 sec$ and $A = 1$ unit area

$$k = \frac{Q}{(T_1 - T_2)} 2\lambda \quad \cdots \quad (1)$$

Let us assume that there is equal probability for the electrons to move in all the six directions

Each electrons travels with thermal velocity 'v' and 'n' is the free electron density then on average of $1/6\ nv$ electron will travel in any one direction.

The number of electrons carrying unit area per unit time from A to B.

$$= \frac{1}{6} nv \quad \cdots \quad (2)$$

According to kinetic theory of gas an average kinetic energy of electron at hot end A of temperature.

$$T_1 = \frac{1}{2} mv^2 = \frac{3}{2} k_B T_1 \quad \cdots \quad (3)$$

where, k_B is Boltzmann's constant

T_1 is the temperature at end A.

Average kinetic energy of electron at cold end B of temperature. $T_2 = \frac{3}{2} k_B T_2 \quad \cdots \quad (4)$

The heat energy transferred per unit area per unit time from end A to B across C $\Bigg\}$ = $\Bigg\{$ Number of electrons × Average K.E of electron moving from A to B $\Bigg\}$

$$Q = \frac{1}{6} nv \frac{3}{2} k_B T_1 \quad \ldots \quad (5)$$

$$= \frac{1}{4} nv k_B T_1$$

Similarly, the deficiency of energy carried from B to A for unit area in the unit time.

$$= \frac{1}{6} nv \frac{3}{2} k_B T_2 \quad \ldots \quad (6)$$

$$= \frac{1}{4} nv k_B T_2 \quad \ldots \quad (7)$$

Hence, net amount of heat energy transferred from A to B per unit area per unit time.

$$Q = \frac{1}{4} nv k_B T_1 - \frac{1}{4} nv k_B T_2$$

$$Q = \frac{1}{4} nv k_B (T_1 - T_2) \quad \ldots \quad (8)$$

Substitute the eqn (8) in eqn (4)

we know that the thermal conductivity

$$k = \frac{1}{4} \frac{nv \, k_B (T_1 - T_2) \, 2\lambda}{(T_1 - T_2)} \quad (\because k_B = k)$$

$$k = \frac{1}{2} nv k \lambda \quad \ldots \quad (9)$$

For metals, Relaxation time (τ) = collision time (τ_c)

So, that $\quad \tau = \tau_c = \frac{\lambda}{v}$

$$\tau v = \lambda \quad \ldots \quad (10)$$

Substitute the eqn (10) in eqn (9)

The thermal conductivity of metal is given by,

$$k = \frac{1}{2} n v^2 k \tau \quad \ldots (11)$$

Eqn (11) is the classical expression for thermal conductivity of metal based on classical free electron theory.

3. Degenerate states:-

Different wave functions with three different quantum numbers may have the same energy value. such a condition is known as degeneracy. for example three independent states having quantum numbers (112), (121) and (211) have the same energy.

For several combinations of quantum number if we have same eigen value and same eigen function then such states and energy levels are called Non degenerate state.

$n_x = 2,\ n_y = 2$ and $n_z = 2$.

$$E_{222} = \frac{12 h^2}{8ma^2} \Rightarrow$$

$$\psi_{222} = \sqrt{\frac{8}{a^3}} \sin \frac{2\pi x}{a} \sin \frac{2\pi y}{a} \sin \frac{2\pi z}{a}$$

Thus four quantum numbers are needed to define a quantum state of an electron completely.

4. Fermi - Dirac Statistics:-

Fermi distribution function gives the distribution electron among the various energy levels are function temperature.

It is given by $F(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} \quad \ldots (1)$

where,

$E \rightarrow$ Energy of the level whose occupancy is being considered.

$E_F \rightarrow$ Energy of the fermi level

$k_B \rightarrow$ Boltzmann's constant

$T \rightarrow$ Absolute temperature

Effect of Temperature on fermi function:

The effect of temperature in fermi function $F(E)$ can be discussed with respect to eqn.

$$F(E) = \frac{1}{1+e^{(E-E_F)/k_B T}} \quad \dots (2)$$

case (i): when $T = 0k$ and $E < E_F$

When $T = 0k$ and $E < E_F$, then applying the values in the expression.

$$F(E) = \frac{1}{1+e^{(E-E_F)/k_B T}}$$

$$F(E) = \frac{1}{1+e^{(-ve/0)}} \qquad \left[\because e^{-\infty} = 0\right]$$

$$F(E) = \frac{1}{1+e^{(-\infty)}} = \frac{1}{1+0} = 1$$

$$F(E) = 1$$

Thus, there is 100% chance for the electrons to occupy the energy levels below the fermi level.

case (ii): At $T = 0k$ and $E > E_F$

$$F(E) = \frac{1}{1+e^{(E-E_F)/k_B T}}$$

$$F(E) = \frac{1}{1+e^{(+ve/0)}} \qquad \left[\because e^{+\infty} = \infty\right]$$

$$F(E) = \frac{1}{1+e^{(+\infty)}} = \frac{1}{1+\infty} = 0.$$

$$= 1/\infty \quad F(E) = 0.$$

Thus, there is 0% chance for the electrons to occupy energy level above the fermi energy level.

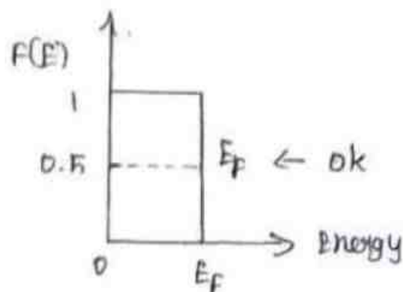
case (iii) $T > 0$ k and $E = E_F$

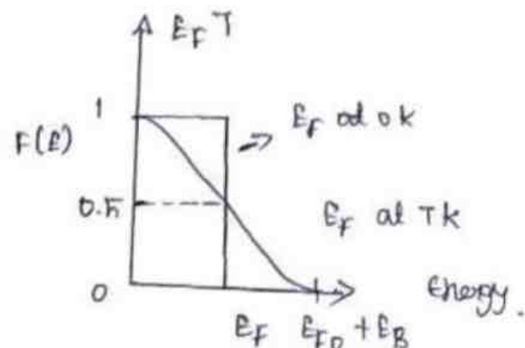
$$F(E) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

$$\left[\because e^0 = 1\right]$$

$$F(E) = \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

there is 50% chance for the electrons to be filled and not to be filled within the fermi energy level.





Fermi level, fermi energy & their Importance :-

There are defined as the highest energy level filled by the electrons in that energy level with higher energy values.

Fermi level :- The fermi level is the highest reference energy level of a particle at absolute zero.

Importance :- It is the reference energy level which separates the filled energy levels and vacant energy levels.

Fermi energy (E_F). The fermi energy is the maximum energy of the quantum state corresponding to fermi energy level at absolute zero.

Importance : Fermi energy determines the energy of the particle at any temperature.

⑧

Applications of Fermi - Dirac statistics :-

* The collection of these free electrons forms a sort of gas known as fermi gas.
* As the temperature of the system is decreased, the energy the system also decreases.
* The electrons tend to occupy lower energy states as the system is cooled.

5. Density of energy states :-

Definition :-

Density of state $Z(E)\,dE$ is defined as the number of energy states present per unit volume of a metal in an energy interval E and $E+dE$.

$$\text{Density of states } Z(E)\,dE = \frac{\text{Number of Energy state available between energy interval } E \text{ \& } E+dE \text{ in metal piece } N(E)\,dE}{\text{volume of that metal piece } (V)}$$

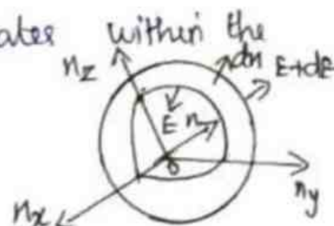
Derivation :-

Let us consider a cubical sample with side 'a', A sphere is constructed with three quantum numbers n_x, n_y, n_z as coordinate axes in three- dimensional space.

Therefore the number of energy states within a sphere of radius.

$$= \left[\frac{4}{3}\pi n^3\right] \quad \cdots\cdots (1)$$

The number of available energy states within the sphere of radius 'n' due to one octant.



$$= \frac{1}{8}\left[\frac{4}{3}\pi n^3\right] \quad \cdots (2)$$

that sphere 1/8 of its volume will satisfy this condition.

Similarly the number of available energy states within the sphere of radius $n+dn$ corresponding energy.

⑨

$$= \frac{1}{8}\left[\frac{4}{3}\pi (n+dn)^3\right] \quad \cdots \quad (3)$$

Therefore, number of available energy state b/w the sphere of radius 'n' and 'n+dn' is,

$$N(E)\,dE = \frac{1}{8}\left[\frac{4}{3}\pi (n+dn)^3\right] - \frac{1}{8}\left[\frac{4}{3}\pi (n^3)\right]$$

$(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

$$= \frac{1}{8}\left[\frac{4}{3}\pi\right]\left(\overset{(a+b)^3}{\underset{a\ \cdot\ b}{(n+dn)^3}} - (n^3)\right)$$

$$= \frac{1}{8}\left[\frac{4}{3}\pi\right]\left(n^3 + dn^3 + 3n^2 dn + 3n\,dn^2 - n^3\right)$$

$$= \frac{\pi}{6}\left[n^3 + dn^3 + 3n^2 dn + 3n\,dn^2 - n^3\right]$$

since the values 'dn' is very small, and dn^3, dn^2 is very large then this terms are neglected.

$$N(E)\,dE = \frac{\pi}{6}\,3n^2\,dn \quad ---- \quad (4)$$

Number of available energy states between the energy interval and $E+dE$ is given by,

$$N(E)\,dE = \frac{\pi}{2}\,n^2\,dn$$

$$N(E)\,dE = \frac{\pi}{2}\,n\,n\,dn \quad \cdots\cdots \quad (5)$$

we know that, the energy of an electron in a cubical metal piece of side 'a' is,

$$E = \frac{n^2h^2}{8ma^2} \longrightarrow (6)$$

$n^2 = (n_x^2 + n_y^2 + n_z^2)$

$$n^2 = \frac{8ma^2E}{h^2} \longrightarrow (7)$$

Taking square on both sides.

(10)

$$n = \left[\frac{8ma^2 E}{h^2}\right]^{1/2} \longrightarrow (8)$$

by differentiating eqn (7), we get

$$2ndn = \frac{8ma^2\, dE}{h^2}$$

$$ndn = \frac{8ma^2\, dE}{2h^2} \longrightarrow (9)$$

By substituting eqn's (8) and (9) in eqn (5).

$$N(E)\,dE = \frac{\pi}{2}\, n\, ndn$$

$$N(E)\,dE = \frac{\pi}{2}\left[\frac{8ma^2 E}{h^2}\right]^{1/2}\left(\frac{8ma^2\, dE}{2h^2}\right)$$

$$N(E)\,dE = \frac{\pi}{2}\times\frac{1}{2}\left[\frac{8ma^2 E}{h^2}\right]^{1/2}\left(\frac{8ma^2\, dE}{h^2}\right)$$

$$= \frac{\pi}{4}\left[\frac{8ma^2}{h^2}\right]^{1/2}(E)^{1/2}\left(\frac{8ma^2}{h^2}\right)^{1}(dE)$$

$$N(E)\,dE = \frac{\pi}{4}\left[\frac{8ma^2}{h^2}\right]^{3/2} E^{1/2}\, dE \longrightarrow (10) \quad \because \left(\frac{1}{2}+1,\ \frac{2+1}{2}=\frac{3}{2}\right)$$

According to pauli's exclusion principle, in each state two electrons can occupy ~~each state~~.

Therefore, number of filled energy states available for electron occupancy is given by.

$$N(E)\,dE = 2\times\frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2} E^{1/2}\, dE \quad \because \left(2\times 3/2,\ \frac{3\times 2}{2}\right)$$

$$= \frac{\pi}{2}\,\frac{(8m)^{3/2}\,(a^2)^{3/2}}{(h^2)^{3/2}}\,(E)^{1/2}\, dE$$

$$= \frac{\pi}{2}\,(8m)^{3/2}\left(\frac{a}{h}\right)^{3} E^{1/2}\, dE$$

(11)

$$(8m)^{3/2} \Rightarrow (8m)^1 (8m)^{1/2}$$

$$= 4\times(2m)^1 (4\times 2m)^{1/2} \quad \because (\sqrt{4} = 2)$$

$$= 4\times 2 \times(2m)^1 (2m)^{1/2}$$

$$= 8\times(2m)^1 \times (2m)^{1/2} \Rightarrow 8\times(2m)^{3/2}$$

$$N(E)\,dE = \frac{\pi}{2}\left(\frac{a}{h}\right)^3 \times 8(2m)^{3/2} E^{1/2}\,dE$$

$$\boxed{N(E)\,dE = \frac{4\pi a^3}{h^3}\times(2m)^{3/2} E^{1/2}\,dE} \longrightarrow (11)$$

Density of states = Number of energy states per unit volume in the energy interval E and E+dE.

$$\frac{N(E)\,dE}{V} = Z(E)\,dE$$

$$Z(E)\,dE = \frac{N(E)\,dE}{V} = \frac{\frac{4\pi a^3}{h^3}\times(2m)^{3/2} E^{1/2}\,dE}{a^3} \longrightarrow (12)$$

Since $V = a^3 = 1$, the volume of the metal.

Therefore, Density of states,

$$\boxed{Z(E)\,dE = \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2}\,dE} \longrightarrow (13)$$

Eqn (13) represents the density of states.

This is the expression for the density of charge carriers in the energy interval E and E+dE.

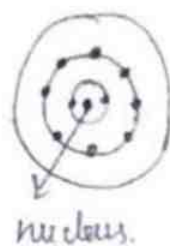
It is used to calculate carrier concentration in metals and semiconductors.

6- Classical free electron theory:-

(i) free electrons in metals:-

* To understand the origin of free electrons in metals, Let us consider the particular case of Sodium Metal.

* The electron in the outermost orbit (3s) is responsible for most of the chemical properties of Sodium and it is called valence electron



* The radius of the outermost orbit is nearly 1.9Å

* The free electron theory of solids explains the structure and properties of solids through their electronic structure.

Main stages of free electron Theory of solids:-

1. classical free electron theory (Drude and Lorentz free electron theory)

* This theory was proposed by drude and Lorentz in the year 1990. According to this theory, the free electrons are mainly responsible for electrical conduction in metals. This theory obeys the laws of classical mechanics.

* Here, the free electrons are assumed to move in a constant potential.

2. Quantum free electron theory (Sommerfeld Quantum theory)

* This theory was proposed by Sommerfeld in the year 1928. and,

* This theory obeys the laws of quantum mechanics.

* According to this theory, the electrons in a metal move in a constant potential.

5. Zone theory or Band theory of solids:-

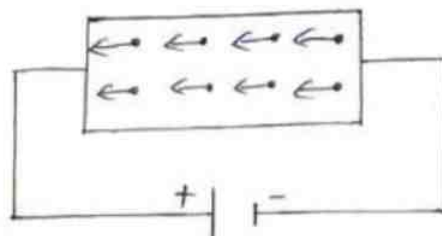
* Bloch proposed this theory in the year 1928. According to this theory, the free electrons move in a periodic potential.

* This theory explains electrical conductivity based on the energy bands.

ii) Assumption (postulates) of classical free Electron theory:-

* A solid metal has nucleus with revolving electrons. The electrons move freely like molecules in a gas.

* The free electrons move in a uniform potential field due to the ions fixed in the lattice.



* In the absence of electric field (E=0), the free electrons move in random directions and collide with each other.

* During this collision no loss of energy is observed since the collisions are elastic.

* When the presence of electric field (E≠0) the free electrons are accelerated in the direction opposite to the direction of applied electric field.

* Since the electrons are assumed to be perfect gas, they obey the laws of classical theory of gases.

* classical free electrons in the metal obey Maxwell-Boltzmann statistics.

* Therefore, free electrons can be assigned with mean free path, mean collision time and average speed.

iii) Basic Definitions:-

1. Drift velocity (V_d)

The drift velocity is defined as the average velocity acquired by the free electron in particular direction. due to the applied electric field.

$$\text{Drift velocity} = \frac{\text{Average distance travelled by the electron}}{\text{Time taken}}$$

$$V_d = \frac{\lambda}{\tau} \, ms^{-1}$$

2. Mobility (μ)

The mobility is defined as the drift velocity (V_d) acquired by the electron per unit electric field (E).

$$\mu = \frac{V_d}{E} \, m^2 V^{-1} s^{-1}$$

3. Mean free path (λ)

The average distance travelled by a electron b/w two successive collision is called mean free path.

4. Mean collision time (τ_c) (or) collision time:-

It is the time taken by the free electron b/w two successive collisions. $\tau_c = \frac{\lambda}{V_d}$ sec

5. Relaxation time (τ)

It is the time taken by the electron to reach

(15)

equilibrium position from disturbed position in the presence of electric field. $\tau = \frac{l}{V_d}$ sec.

where, l is the distance travelled by the electron. The value of relaxation time is of the order of 10^{-14} sec.

6. Band gap (E_g) :-

Band gap is the energy difference b/w in the minimum energy of conduction band and the maximum energy of valence band.

7. Current density (J) :-

current density (J) is defined as the flow of current per unit area of cross section of current carrying conductor. $J = \frac{I}{A}$ Am^{-2}

8. Electrical conductivity (σ) :-

The amount of electrical charges conducted (Q) per unit time across unit area (A) of a solid per unit applied electrical field (E) is called the electrical conductivity

It is denoted by σ, $\sigma = \frac{Q}{tAE}$ $\Omega^{-1} m^{-1}$

$$\sigma = \frac{J}{E} \Omega^{-1} m^{-1}$$

$\left(\because J = \frac{Q}{tA}\right)$

where, 'J' is the current density. $J = \sigma E$.

7. Energy Bands in solids:-

1. Formation of Energy Bands in solids:-

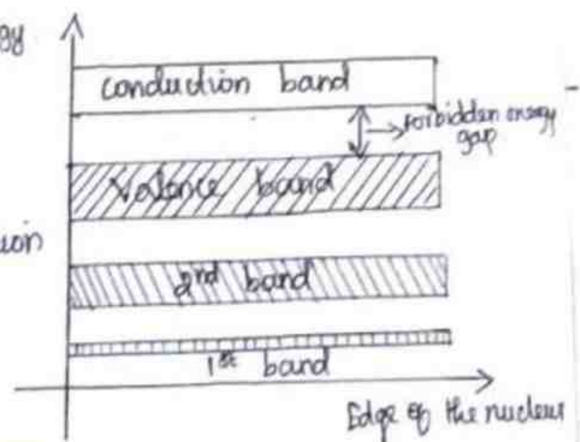
* The solid crystals are formed when the isolated atoms are brought together various interactions occur b/w the nighboring atoms.

* At particular inter-atomic spacing 'd' there is a properer balance b/w forces of attraction to form a crystal.

* In case of a single atom, there is a single energy for an electron orbit.

* As a result, the number of permissible energy levels is formed which is called an energy band.

* Three bands are important from the conductivity point view which are, valence band, conduction band forbidden gap or band gap.



Valence band:-

* Grouping of energy levels of valence electrons.

* have lower energy than conduction band

* Electrons are loosely band.

Conduction band:-

* Grouping of conduction electrons.

* have a higher energy than valance band.

Forbidden band:-

* Separation b/w valence and conduction band.

* Energy associated with forbidden ban.

* Unit → electron volt (ev) $1eV = 1.6 \times 10^{-19} J$.

2. Classifications on the basis of energy band theory

Based on the ability of various materials to conduct current, the materials are classified as conductors, insulators and the semiconductors.

1 Conductors:-

* A material having large number of free electrons can conduct very easily.

* For example, copper has 8.5×10^{28} free electrons per cubic meter which is a very large number.

* Hence copper is called good conductor.

* Infact, in the metals like copper, aluminium there is no forbidden gap b/w valence band and conduction band.

* The two bands overlap. Hence even at room temperature a large number of electrons are available for conduction.

2. Insulators:-

* An insulator is a material having extremely poor electrical conductivity.

* In case of such insulating material, there exists a large forbidden gap in b/w the conduction band and the valence band.

* Practically it is impossible for an electron to jump from the valence band to the conduction band.

* Hence such materials cannot conduct and called insulators.

* The forbidden gap is very wide, approximately of about 7eV is present in insulators.

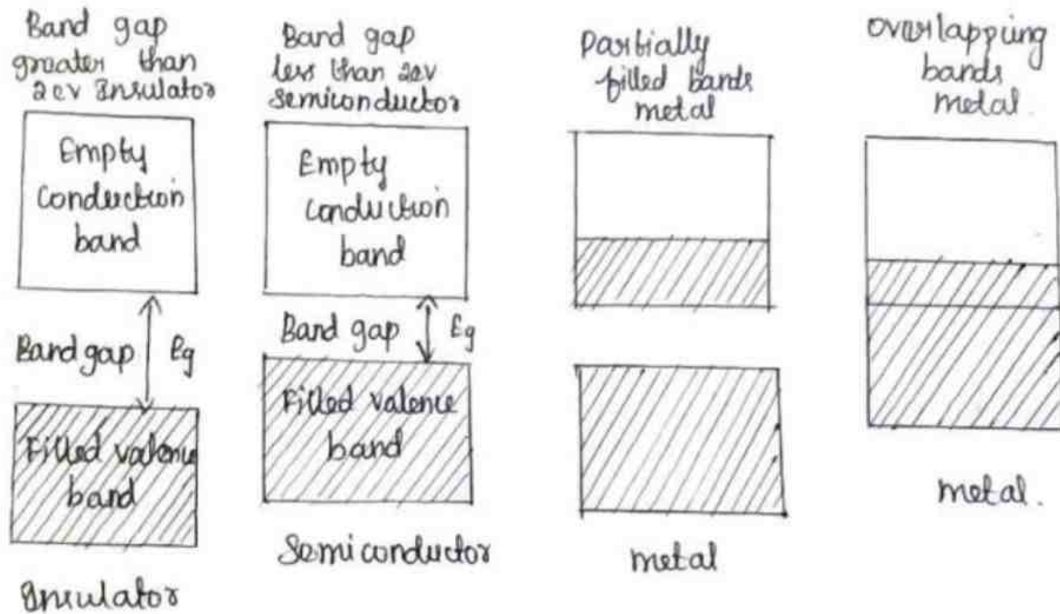
* Example, Glass, Diamond, wood, mica.

3. Semiconductor:-

* Semiconductor is a material which has electrical

properties lying b/w insulator and conductor.

* At low temperature (ex 0°k), it acts as an insulator.
* At ~~room~~ room temperature, it can acts a a conductor.
* Forbidden energy gap is narrow (≈1ev).
* conductivity is proportional to the ~~conc~~ concentration of free electrons. Ex, silicon.



8. Electron Effective mass and concept of Hole:-

when an electron in a periodic potential is accelerated relative to the lattice in an electric field or magnetic field then the mass of that electron is called effective mass.

consider an electron of charge 'q' and mass 'm' acted on by an electric field $\mathcal{E}$ inside the crystal lattice.

the acceleration $a = \frac{q\mathcal{E}}{m}$. since 'a' is not a constan

$\therefore a = \frac{q\mathcal{E}}{m^*}$

mass of the electron 'm' is replaced by its effective mass m^*

The electrical force on the electron $F = m^* a$.

$K \rightarrow$ wave vector.

8. Microscopic classification of magnetic materials:-

The magnetic materials are classified into two categories.

1. The materials without permanent magnetic moment

   Ex:- Diamagnetic materials.

2. The materials with permanent magnetic moment

   Ex: * Paramagnetic materials
   * Ferromagnetic materials
   * Anti-ferromagnetic materials.

1. Diamagnetic materials:-

Definition:-

* In a diamagnetic material the electron orbits are randomly oriented and the orbital magnetic moments get cancelled.

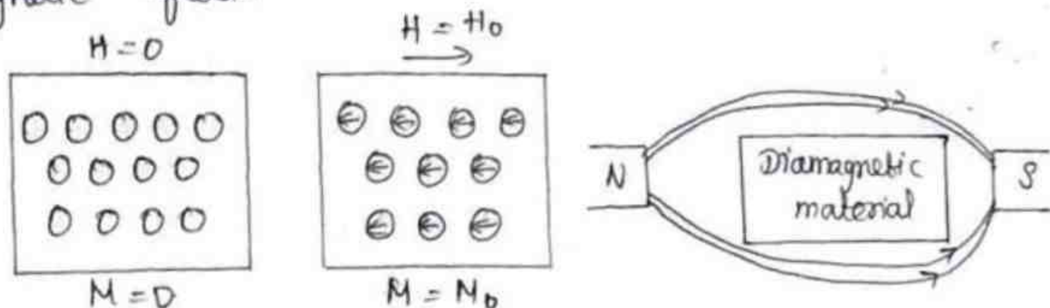
* Similarly, all the spin moments are paired.

* having even number of electrons.

* Therefore, the electrons are spinning in two opposite directions and hence the net magnetic moment is zero.

* When an external magnetic field is applied, the electrons re-orient and align perpendicular to the applied field.

* Their magnetic moment opposes the external magnetic field.



(20)

In the above diagram, there is no penetration of magnetic lines through the diamagnetic material.

Properties:-

* They repel the magnetic lines of force, if placed in a magnetic field.

* The susceptibility is negative and it is independent to temperature and applied field strength ($\chi = -ve$)

* The permeability is less than one (<1)

* there is no permanent dipole moment.

* It has superconducting property.

Example:- Gold, germanium, silicon, antimony, bismuth.

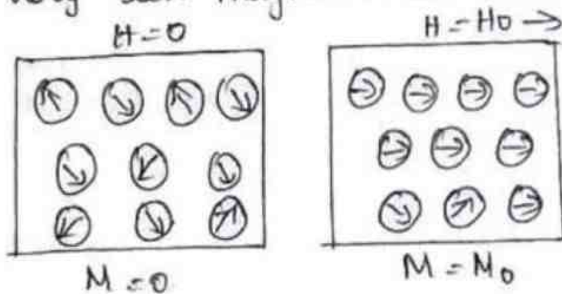
Paramagnetic materials:-

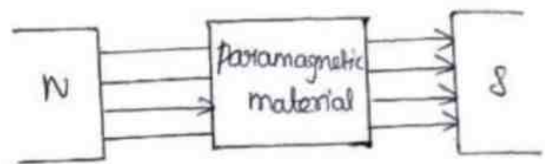
Definition:-

* Paramagnetism is due to the presence of few unpaired electrons which gives rise to the spin magnetic moment.

* In the absence of external magnetic field.

* The magnetic moments are randomly oriented and possess very less magnetization in it.





Effect of magnetic field:-

when an external magnetic field is applied to paramagnetism material, the magnetic moments align themselves along the field direction and the material is said to be magnetized. This effect is known as paramagnetism.

$$\chi \propto \frac{1}{T} \text{ Then } \chi = \frac{C}{T}$$

This is known as the curie law of paramagnetism and C is a constant called curie constant.

Properties:-

* The magnetic lines of force pass through the materials.
* The susceptibility is positive and is given by $\chi = \frac{C}{T-\theta}$
* The permeability is greater than one (>1)
* There is a permanent magnetic moment.
* When the temperature is less than the curie temperature, paramagnetic materials become diamagnetic materials
* Spin alignment is random in nature.

Example:- platinum, $CuSO_4$, $MnSO_4$, Aluminium, etc.

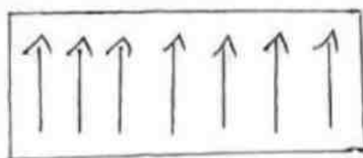
Ferromagnetic materials:-

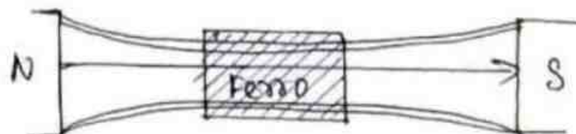
Definition:-

* Ferromagnetism is due to the presence of more unpaired electrons.
* Even in the absence of external field.
* The magnetic moments align parallel to each other.
* So that it has large magnetism. This is called spontaneous magnetization.

Effect of magnetic field:-

* This indicates that there is a strong internal field within the material which makes the atomic magnetic moments align with each other.
* This phenomenon is known as ferromagnetism.





If a small external magnetic field is applied the magnetic moments align in the field direction and become very strong magnets.

Properties of ferromagnetic materials:-

* All the magnetic lines of force pass through the material.
* Its susceptibility is high positive and depends on the temperature. It is given by $\chi = \frac{C}{T-\theta}$
* The permeability is very much greater than one
* They have enormous permanent dipole moment.
* When the temperature is greater than the curie temperature, the ferromagnetic material becomes paramagnetic material.
* The ferromagnetic material has equal magnitude dipole lying parallel to each other.
* They exhibit hysteresis curve.

Example:- Nickel, iron, steel, etc,

9. Effective Mass of Electron and Hole :-

The mass acquired by an electron when it is accelerated in a periodic potential is called effective mass of an electron. It is denoted by m^*.

Explanation :-

Varies with respect to the field applied. This varying mass is called effective mass (m^*)

Derivation of effective mass of electron :-

Group velocity with which the electron can travel

$$v_g = \frac{d\omega}{dk} \quad \ldots (1)$$

where ω angular frequency of the electron

$k \rightarrow$ wave vector

$$\left[\because \omega = 2\pi\nu \quad \nu = \frac{\omega}{2\pi} \quad \hbar = \frac{h}{2\pi} \right]$$

we know that, $E = h\nu$

$$E = \frac{h\omega}{2\pi}$$

$$E = \hbar\omega$$

$$\omega = \frac{E}{\hbar} \quad \ldots (2)$$

Substituting (2) in (1)

$$v_g = \frac{d}{dk}\left(\frac{E}{\hbar}\right)$$

$$v_g = \frac{1}{\hbar}\frac{dE}{dk} \quad \ldots (3)$$

The acceleration 'a' experienced by the electron is given by,

$$a = \frac{d(v_g)}{dt}$$

$$= \frac{d}{dt}\left(\frac{1}{\hbar}\left(\frac{dE}{dk}\right)\right)$$

$$= \frac{1}{\hbar}\frac{d^2E}{dk^2} \cdot \frac{dk}{dt} \quad \ldots (4)$$

Momentum (p) of an electron inside the crystal.

$$p = \frac{h}{\lambda} \text{ (from De-Broglie's eqn)}$$

$\div$ and $\times$ in 2π,

$$\because \left(\hbar = \frac{h}{2\pi}\right)$$

$$2\pi \times \frac{p}{2\pi} = \frac{h}{2\pi} \frac{2\pi}{\lambda} \qquad \because k = \frac{2\pi}{\lambda}$$

$$p = \hbar k \quad \ldots\ldots (5)$$

Differentiating the eqn (5) with respect to 't'.

$$\frac{dp}{dt} = \hbar \frac{dk}{dt} \qquad \left(\because F = \frac{dp}{dt}\right)$$

$$F = \hbar \frac{dk}{dt}$$

$$\frac{dk}{dt} = \frac{F}{\hbar} \quad \ldots\ldots (6)$$

Substituting eqn (6) in (4)

$$a = \frac{1}{\hbar} \frac{d^2E}{dk^2} \cdot \frac{F}{\hbar}$$

$$= \frac{1}{\hbar^2} \frac{d^2E}{dk^2} \cdot F$$

$$F = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}\right] a \quad \ldots (7)$$

acceleration of the electron due to this field,

$$a = \frac{eE}{m^*} = \frac{F}{m^*} \qquad (\because F = eE)$$

(or) $$a = \frac{F}{m^*}$$

$$F = m^* a$$

comparing the eqn (7) & (8) we have

$$m^* \cancel{a} = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}\right] \cancel{a}$$

$$\therefore m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$$

The above eqn indicates that the effective mass of an electron is not a constant, but depends on the value of $\frac{d^2E}{dk^2}$

Special cases:-

Case (i):- If $\frac{d^2E}{dk^2}$ is positive, then m^* is also positive

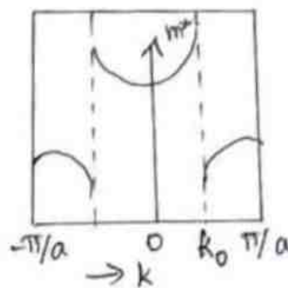
Case (ii):- If $\frac{d^2E}{dk^2}$ is negative, then m^* is also negative.

Case(iii):- If $\frac{d^2E}{dk^2}$ is more, then the electrons behave as light particle.

Case (iv):- If $\frac{d^2E}{dk^2}$ is very small, then the electrons behave as heavy particle.

Variation of m^* with k:-

The point of inflection, m^* is negative and as k tends to $\frac{\pi}{a}$, it decreases to a small negative value.



10. Quantum free Electron theory:-

This theory uses quantum concepts and hence it is known as quantum free electron theory.

He approached the problem quantum mechanically using fermi-Dirac statistics instead of classical maxwell Boltzmann statistics.

Postulates of Quatum free electron theory:-

* The potential energy of an electron is uniform or constant within the metal.

* The electrons have wave nature.

* The allowed energy levels of an electron are quantized.

* The free electrons obey fermi - Dirac statistics.

Merits of Quantum free electron theory:-

* This theory treats the electron ~~quatum~~ quantum mechanically rather than classically.

* It explains the electrical conductivity, thermal conductivity, specific heat capacity of metals, Photoelectric effect and compton effect, etc.

Demerits of Quantum free electron Theory:-

* It also fails to explain the positive value of Hall coefficient and some of the transport properties of the metals.

Electrons in Metals - particle in a three dimensional box:-

* The solution of one - dimensional potential well is extended for a three - dimensional potential box.

* We have to use three quantum numbers, n_x, n_y and n_z, corresponding to the three coordinate axes namely x, y and z respectively.

Energy of the particle = $E_x + E_y + E_z$

$$E_{n_x n_y n_z} = \frac{n_x^2 h^2}{8ma^2} + \frac{n_y^2 h^2}{8mb^2} + \frac{n_z^2 h^2}{8mc^2}$$

If $a = b = c$ as for a cubical box, then

$$E_{n_x n_y n_z} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2 + n_z^2\right] \quad \ldots (1)$$

The corresponding normalised wave function of an electron in a cubical box may be written as,

$$\psi_{n_x n_y n_z} = \sqrt{\frac{2}{a} \times \frac{2}{a} \times \frac{2}{a}} \sin \frac{n_x \pi x}{a} \sin \frac{n_y \pi y}{a} \sin \frac{n_z \pi z}{a}$$

$$\psi_{n_x n_y n_z} = \sqrt{\frac{8}{a^3}} \sin \frac{n_x \pi x}{a} \sin \frac{n_y \pi y}{a} \sin \frac{n_z \pi z}{a} \quad \ldots (2)$$

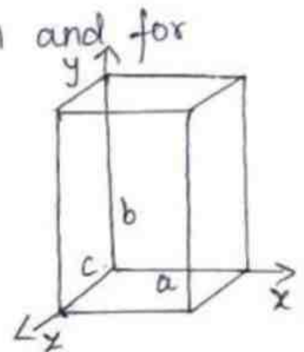
from the equations (1) and (2), we understand that several combinations of the three quantum numbers lead to different energy eigen values and eigen functions.

Example:-

$$n_x = 1, \; n_y = 1, \; n_z = 2$$

$$n_x^2 + n_y^2 + n_z^2 = 6$$

Similarly, for a combination $n_x = 1, n_y = 2, n_z = 1$ and for a combination $n_x = 2, n_y = 1, n_z = 1$.



$$n_x^2 + n_y^2 + n_z^2 = 6$$

$$\therefore E_{112} = E_{121} = E_{211} = \frac{6h^2}{8ma^2} \quad \ldots (3)$$

The corresponding wave functions are written as

$$\left.\begin{aligned} \psi_{112} &= \sqrt{\frac{8}{a^3}} \sin \frac{\pi x}{a} \sin \frac{\pi y}{a} \sin \frac{2\pi z}{a} \\ \psi_{121} &= \sqrt{\frac{8}{a^3}} \sin \frac{\pi x}{a} \sin \frac{2\pi y}{a} \sin \frac{\pi z}{a} \\ \psi_{211} &= \sqrt{\frac{8}{a^3}} \sin \frac{2\pi x}{a} \sin \frac{\pi y}{a} \sin \frac{\pi z}{a} \end{aligned}\right\} \quad \ldots (4)$$

11. Tunneling :-

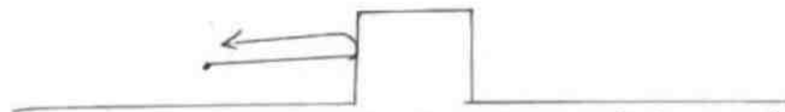
* According to classical ideas, a particle striking a hard well has no chance of leaking through it. But, the Behaviour of a quantum particle is different due to the wave nature associated with it.

* we know that when an electromagnetic wave strikes at the interface of two media, it is partly reflected and partly transmitted through the interface and enters the second medium.

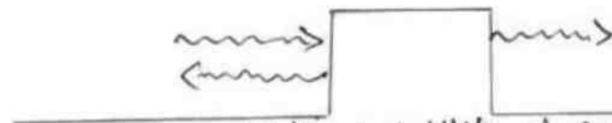
* particle with energy $E < V$ approaching potential barrier of height V.

* For the particle to overcome the potential barrier, it must have an energy equal to or greater than V.

(a) A particle with energy $E < V$ approaching a potential barrier



(b) From classical mechanics, the particle must be reflected by the barrier.



The particle has finite probability of penetrating the barring.

* Quantum mechanics leads to an entirely new result.

* It shows that there is a finite chance for the electron to leak to the other side of the barrier.

* It is noted that the electron tunneled through the potential barrier and hence in quantum mechanics,

This is called tunneling.

The transmission of electrons through the barrier is known as barrier penetration.

12. Paramagnetism in the conduction Electrons in Metals:-

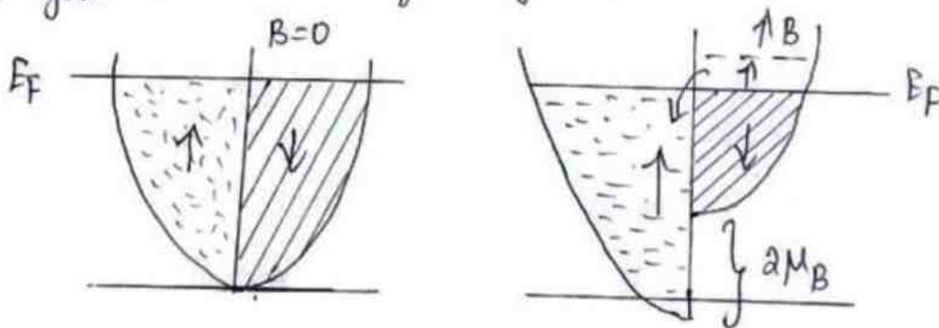
* According to Langevin's theory the paramagnetic susceptibility is inversely proportional to the temperature.

* It was W. Pauli (1927) who demonstrated that this is due to paramagnetism of free electrons.

* Since they can orient only in two directions, either along the magnetic field or against it.

* In the absence of an external magnetic field, the distribution of electrons with spins parallel to z-direction is equal to the number of electrons with opposite spins and hence the net magnetic moment of the electron gas is zero.

* As a result of this, the fermi level for the two spin distributions shift with respect to each other and give rise to energetically unstable situation.



$$N_{eff} = \frac{1}{2} Z(E_F)\mu_B B \quad \cdots (1)$$

where the factor ½ is due to the fact that the density of states of one spin distribution is half of the total density of states. μ_B is magnetic moment of electron.

Since each flip increases the magnetization by $2\mu_B$ the net magnetization is given by.

$$M \approx N_{eff} \times 2\mu_B = Z(E_F)\mu_B^2 B \quad \ldots \ldots (2)$$

hence the pauli spin susceptibility of the electron gas is,

$$\chi_p \approx \mu_0 \mu_B^2 Z(E_F) \quad \ldots \ldots (3) \qquad \left(\because B = \mu_0 H, \ \chi_p = \frac{M}{H} \right)$$

According to equation (3), χ_p is essentially temperature independent.

$$Z(E_F) = \frac{3N}{2E_F}$$

N - No of electrons per unit volume.

eqn (3) becomes,

$$\chi_p = \frac{3N\mu_0\mu_B^2}{2E_F} = \frac{3\mu_0 N\mu_B^2}{2kT_F}$$

where $E_F = kT_F$ this eqn can be rewritten in terms of the classical susceptibility as.

$$\boxed{\chi_p = \frac{3}{2}\chi \frac{1}{T_F}}$$

where $\chi = \dfrac{\mu_0 N\mu_B^2}{k}$

Since T_F is normally very high, χ_p is smaller than χ by about two orders of magnitude, which is in agreement with the experimental results.

In transition metals, the paramagnetic susceptibility, χ_p is exceptionally high, because $Z(E_F)$ is large.

1.8 Exchange Interaction And Ferromagnetism :-

* The ferromagnetic property is exhibited by transition elements such as iron, cobalt, and nickel at room temperature.

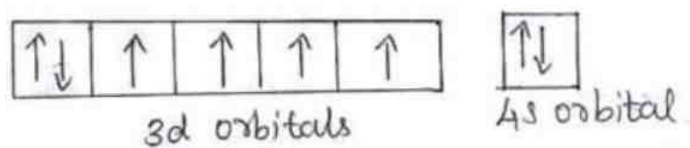
* rare earth elements like gadolinium.

* The ferromagnetic materials possess parallel alignment of dipoles.

* This parallel alignment of dipoles is not due to the magnetic force exiting between any two dipoles.

* The reason is that the magnetic potential energy is very small and it is smaller than thermal energy.

* For iron, the six electrons present in the 3d Subshell occupy the orbitals such that there are four unpaired electrons and two paired electrons.



* these four unpaired electrons contribute a magnetic moment of 4β. This arrangement shows the parallel alignment of four unpaired electrons.

* The pauli's exclusion principle and electrostatic interaction energy are combined together and constitute a new kind of interaction known as exchange interaction.

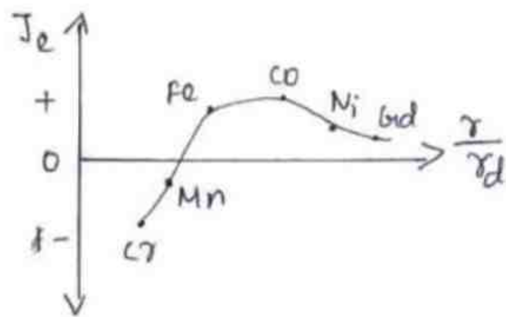
* The exchange interaction is a quantum mechanical concept.

* The exchange interaction between any two atoms is given by.

$$E_{ex} = -J_e S_1 S_2$$

* where J_e is the numerical value of the exchange integral, S_1 and S_2 are the spin angular momenta of the first and second electrons.

* A plot between the exchange integral and the ratio of the interatomic separation to the radius of 3d orbital (r/r_d).



Quantum Interference Devices :-

Squid

SQUID Stands for Superconducting Quantum Interference Device

It is an ultra - sensitive Instrument used to measure very weak magnetic field of the order of 10^{-14} tesla.

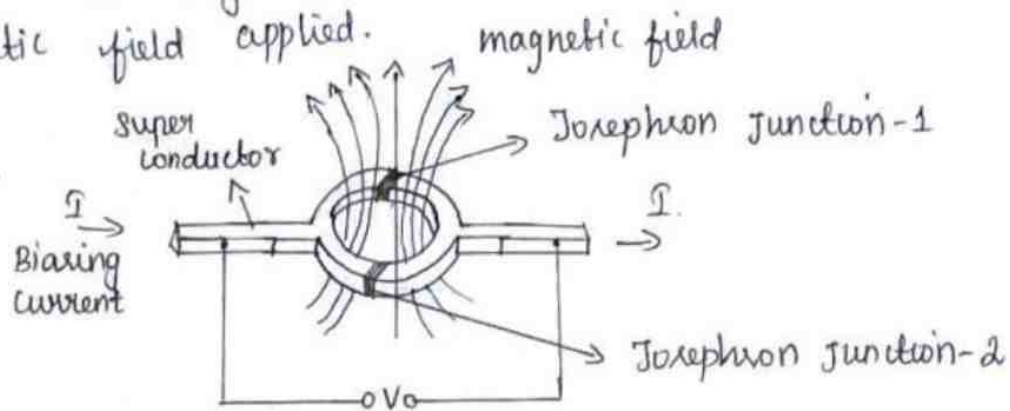
principle :-

We know that a small change in magnetic field produces variation in the quantum flux.

Description and working :-

A SQUID It consists of a superconducting ring which can have magnetic fields of quantum values (1, 2, 3 ....) of flux placed inbetween two Josephson junctions.

* when the magnetic field is applied perpendicular to the plane of the ring,
* The current is induced at the two Josephson Junctions.
* The induced current produces the interference pattern and it flows around the ring so that the magnetic flux in the ring can have the quantum value of magnetic field applied.

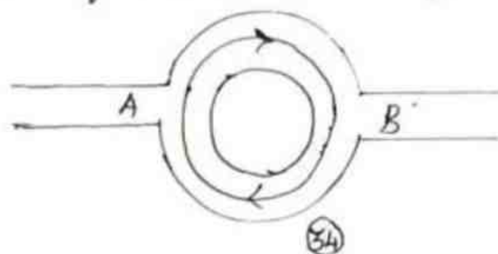


Application:-

* SQUID can be used to detect the variation of very minute magnetic signals in terms of quantum flux.
* It can also be used as storage device for magnetic flux
* SQUID is useful in the study of earthquakes removing paramagnetic impurities, detection of magnetic signals from the brain, heart etc.

Quantum Interference Transistor (QUIT)

Electrons are made to propagate through two arms of the quantum wire ring.



* The wave entering through "A" gets split up into two partial waves.

* A constructive interference can be expected to occur at "B" similar to the optical analogue as they travel through the same distance.

* This device is expected to act as a high-speed transistor.

Magneto resistance:-

Some metallic materials show a large change in resistance on the application of a magnetic field.

This effect is called magneto resistance (MR).

Giant Magneto resistance (GMR)

It is a quantum mechanical magnetoresistance effect observed in multilayers composed of alternating ferromagnetic and non-magnetic conductive layers.

Definition:-

* The effect is observed as a significant change in the electrical resistance depending on whether the magnetization of adjacent ferromagnetic layers are in a parallel or an antiparallel alignment.

* The main application of GMR is magnetic field sensors, which are used to read data in a hard disk devices, biosensors.

* GMR Multilayer structure are also used in magnetoresistive random-access memory (MRAM) as cells that store one bit of information.

Explanation:-

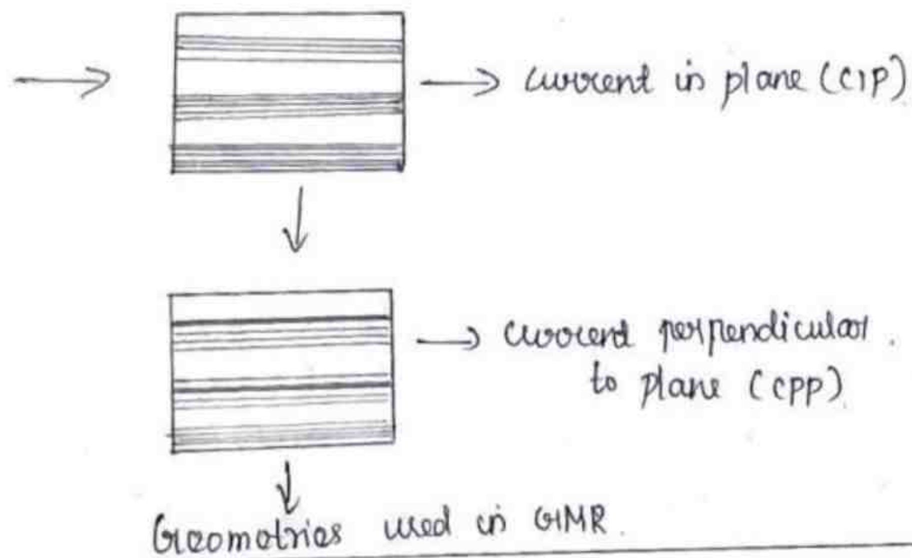
* The GMR is seen in structures which have normal metal and ferromagnetic layers alternatively.

* The electrical conductivity depends on the relative orientation of magnetization in the successive ferromagnetic layers in the stack.

* Two geometries are commonly used in GMR Studies

(i) current in plane of layer and

ii) current perpendicular to plane (CPP) of layers.



Geometries used in GMR.

14. GIANT MAGNETO RESISTANCE (GMR) - Devices Sensor - Spin Valve:-

* A device that works on the principle of the GMR is a spin valve. This device is used in magnetic hard discs for high density dat storage.

* There are 4 layers altogether in a spin valve.

Two ferro magnetic layers are separated by a thin spacer layer.

One ferromagnetic layer is pinned of magnetization

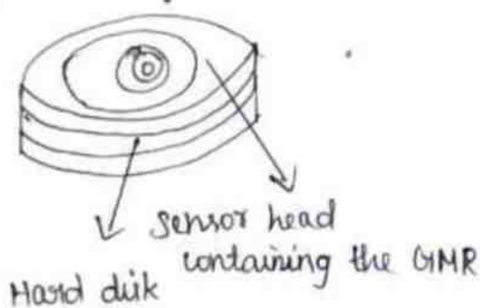
(36)

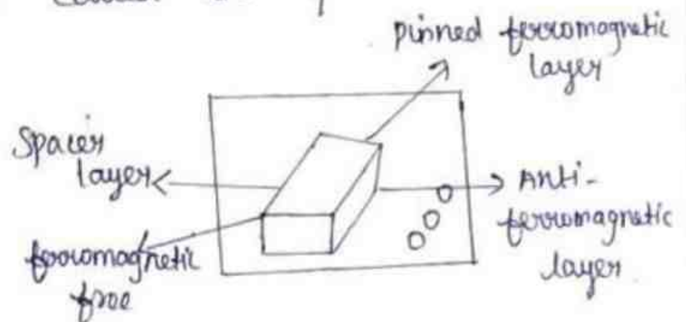
is fixed and is not distributed by changes in field.

* This layer is pinned by adding by a fourth layer, a strong and ferromagnet.

* The other layer, called the free layer, is sensitive to the field produced by the data bit.

* permalloy is usually chosen for both ferromagnetic layers. This structure is called the spin valve.





This generates a significant change in electrical resistance due to the GMR effect.

* As the bit travels under the head, the resistance goes down, the electrons do not scatter very much and the current flow increases.

* As the bit moves on, the resistance increases, the electrons are scattered and the current decreases.

* As the bit travels further from the head, the resistance peaks and the current decreases to its lowest point.

unit-2.

Problems:-

1 Evaluate the fermi function for energy $k_B T$ above the fermi energy.

solution:-

we know fermi function $F(E) = \frac{1}{1+e^{(E-E_F)/K_B T}}$

For an energy $k_B T$ above fermi energy.

$$E - E_F = k_B T.$$

$$F(E) = \frac{1}{1+e^1} = \frac{1}{1+2.7183}.$$

Fermi distribution function $F(E) = 0.2689$.

problem (2)

2. Free electron density of aluminium is $18.1 \times 10^{28} m^{-3}$. calculate its fermi energy at 0K. [planck's constant and mass of free electron are 6.62×10^{-34} Js, and 9.1×10^{-34} kg].

Solution:-

Given data:-

Planck's constant $h = 6.62 \times 10^{-34}$ Js

Mass of electron $m = 9.1 \times 10^{-34}$ kg.

Electron density $N = 18.1 \times 10^{28} m^{-3}$.

fermi energy at 0K.

$$(E_{F_0}) = \left[\frac{3N}{\pi}\right]^{2/3} \frac{h^2}{8m}$$

$$(E_{F_0}) = \left[\frac{3 \times 18.1 \times 10^{28}}{3.14}\right]^{2/3} \left[\frac{(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31}}\right]$$

$$= (1.7292 \times 10^{29})^{2/3} \times 6.019 \times 10^{-38}$$

$$E_{F_0} = 1.8689 \times 10^{-18} J$$

(or) $E_{F_0} = \frac{1.8689 \times 10^{-18}}{1.6 \times 10^{-19}}$ eV

fermi energy at 0K

$E_{F_0} = 11.68$ eV.

3. The fermi temperature of a metal is 24600 K. calculate the fermi velocity.

Solution:

Given data:

Temperature = 24600 K.

The relation b/w Fermi energy, fermi velocity and fermi temperature is given by,

$$E_F = \frac{3}{2} k_B T_F = \frac{1}{2} m V_F^2.$$

$$V_F = \sqrt{\frac{3 k_B T_F}{m}} = \sqrt{\frac{31.38 \times 10^{-23} \times 24600}{9.11 \times 10^{-31}}}$$

Fermi velocity $V_F = 863.30 \times 10^3$ ms^{-1}

4. Use the fermi distribution function to obtain the value of F(E) for $E - E_F = 0.01$ eV at 200K.

Solution:

Given data:

fermi function $F(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$

Boltzmann constant $k_B = 1.38 \times 10^{-23}$ JK^{-1}

$$E - E_F = 0.01 \text{ eV}$$

$$= 0.01 \times 1.6 \times 10^{-19}$$

$$= 1.6 \times 10^{-21} \text{ J}$$

$$T = 200\text{K}$$

$$F(E) = \frac{1}{1+e^{1.6\times10^{-21}/(1.38\times10^{-23}\times200)}}$$

$$= \frac{1}{1+e^{0.5797}} = \frac{1}{1+1.7855} = \frac{1}{2.7855}$$

fermi function = 0.3589.

5. calculate the drift velocity of the free electrons (with mobility of $3.5\times10^{-3}\,m^2V^{-1}s^{-1}$) in copper for an electric field strength of $0.5\,Vm^{-1}$.

Solution:-

mobility $m = 3.5\times10^{-3}\,m^2V^{-1}s^{-1}$

Electric field strength $= 0.5\,Vm^{-1}$

Drifty velocity $V_d = \mu E$

$= 3.5\times10^{-3}\times0.5$

$= 1.75\times10^{-3}\,ms^{-1}$

6. copper has electrical conductivity at 300K as $6.40\times10^{7}\,m^{-1}$. calculate the thermal conductivity of copper. (Lorentz number is $2.44\times10^{-8}\,W k^{-2}$).

Solution.

Mobility $m = 3.5\times10^{-3}\,m^2V^{-1}s^{-1}$

Electric field strength $= 0.5\,Vm^{-1}$

Drift velocity $V_d = \mu E$

$= 3.5\times10^{-3}\times0.5$

$= 1.75\times10^{-3}\,ms^{-1}$.

7. The thermal and electrical conductivities of copper at 20°C are $380\,Wm^{-1}K^{-1}$ and $5.67\times10^{7}\,m^{-1}$ respectively. calculate the Lorentz number.

Solution:-

Electrical conductivity $\rho = 6.40\times10^{7}\,\Omega^{-1}m^{-1}$

$T = 300 k$

$\frac{k}{\sigma} = LT$

$k = \sigma LT$

$k = 6.40 \times 10^{7} \times 2.44 \times 10^{-8} \times 300$

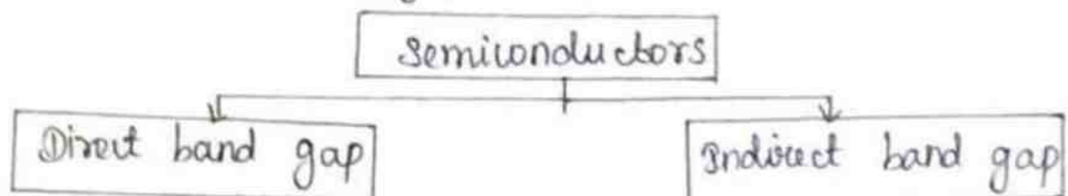
$k = 468.48 \ Wm^{-1} k^{-1}$

unit - III

Semiconductors and Transport physics

1. Direct and indirect Band gap semiconductors:

Semiconductors are also classified into
(a) Direct band gap semiconductor.
(b) Indirect band gap semiconductor



* the electrons and holes in a semiconductor have energy and momentum. the momentum (k) depends on energy (E).

* A plot of energy versus momentum.

* The lower curves represent energy and momentum values of holes in valance band of semiconductors.

* Similarly upper curves denote corresponding values for electrons in conduction band.

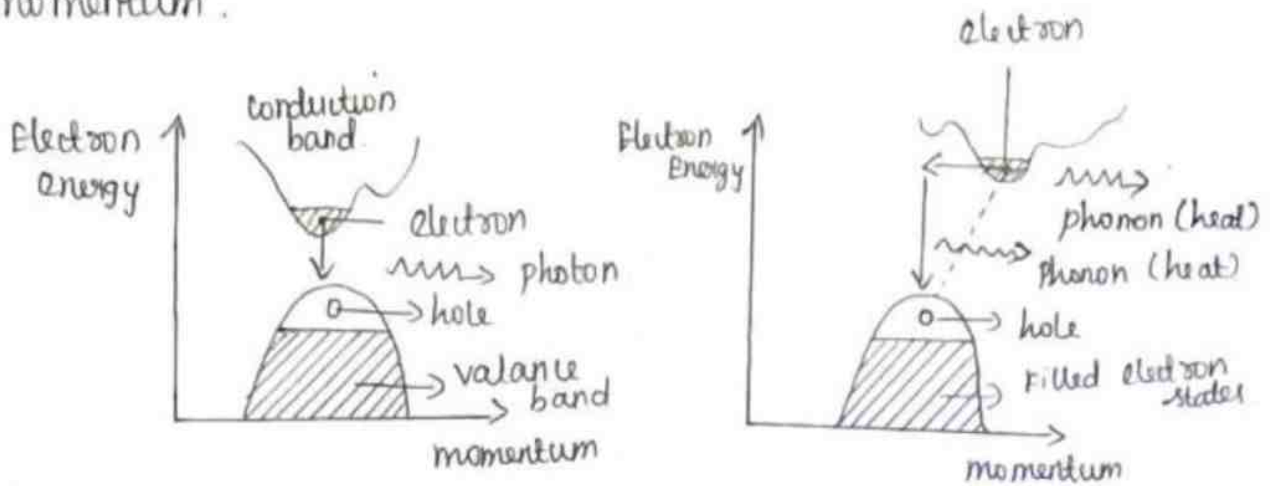
* In direct band gap semiconductor, the energy maximum of valance band and the energy minimum of the conduction band are having same momentum value.

* During the recombination of electron from conduction band with hole in valence band, the momentum of the electrons remains virtually constant.

* The energy equal to band gap energy is released as light photon.

* In the case of indirect band gap semiconductor. The maximum energy of valence band and minimum

conduction band are having different values of momentum.



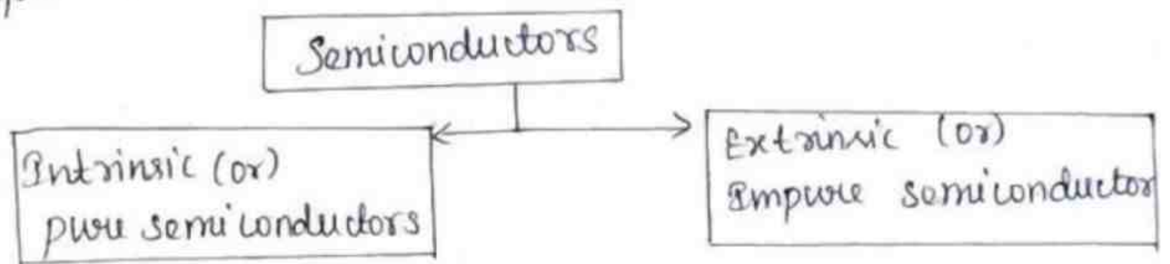
Types of semiconductors:-

* when a suitable impurity is added to a pure Semiconductor its electrical conductivity changes.

* Based on this property, the semiconductors are classified into two types. They are

* Intrinsic semiconductor or pure semiconductor

* Extrinsic Semiconductor or impure Semiconductor or doped semiconductor.



a) Intrinsic Semiconductors:-

* A semiconductor in extremely pure form is known as intrinsic semiconductor.

* Its electrical conductivity changed only by thermal excitation.

* The common examples for intrinsic semiconductors are pure silicon (Si) and (Ge) germanium.

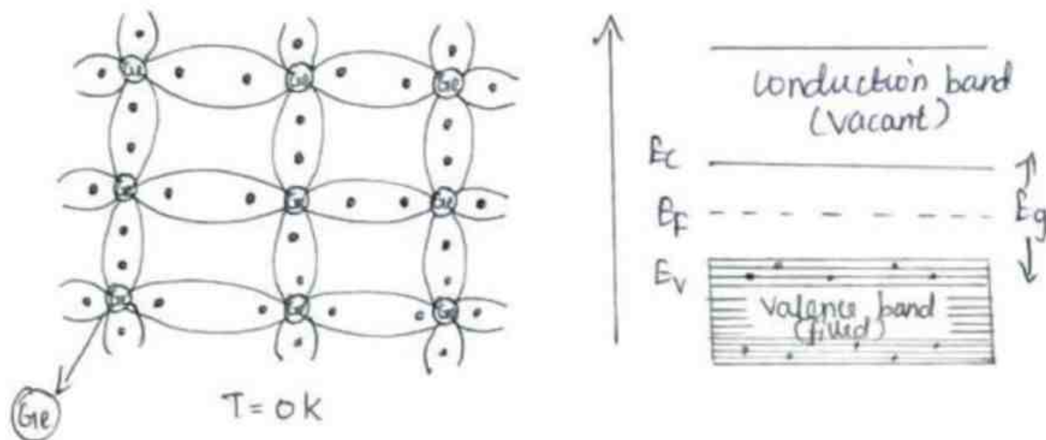
* They belong to fourth group elements in the periodic table.

* Germanium has 32 electrons and silicon has 14 electrons in their atomic structures.

* they are tetravalent atoms since they have four valence electrons. the neighbouring atoms form covalent bands by sharing four electrons with each other so as to form a stable structure

3. Energy band diagram:-

Two - dimensional crystal structure of germanium and energy band representation of intrinsic semiconductor at very low temperature.



* At very low temperature say 0k, no free electrons are available for conduction.

* Hence, this semiconductor behaves as an insulator at very low temperature.

Charge carriers in intrinsic semiconductor:-

when the temperature of intrinsic semiconductor is increased, some of the electrons get sufficient energy to break covalent bonds.

(3)

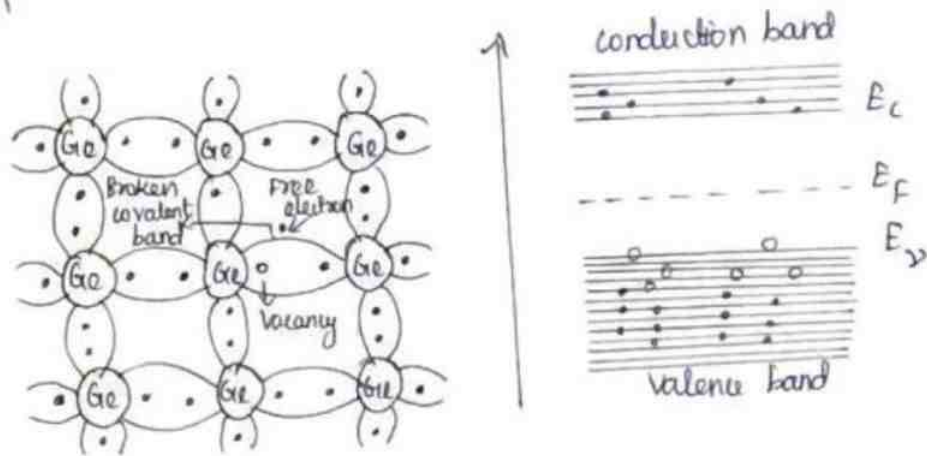
* Once the electrons are liberated from bond, they become free electrons. These free electrons move randomly through crystal.

* The energy required to break a covalent bond and to set an electron free is equal to band gap energy E_g. It is about 0.72 eV for germanium and 1.1 eV for silicon.

* This vacant site is called as a hole. The absence of an electron in covalent bond is known as hole.

* A hole can attract an electron and hence it acts as a positive charge.

* Thus hole will correspondingly move one more step in the direction opposite to the motion of the electron.



Here, the number of electrons is equal to the number of holes at any given temperature.

④

4. Variation of carrier concentration with temperature and impurity.

* In extrinsic semiconductor, the resistivity decreases linearly with increase in temperature.

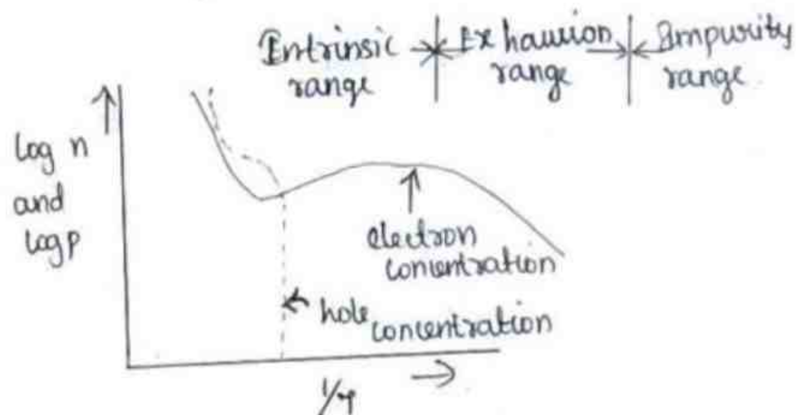
* This variation is considered under three different regions.

  * Extrinsic or impurity range
  * Exhaustion range
  * intrinsic range

For a n-type semiconductor, the variation of carrier concentration n and p with temperature.

* With increase in temperature, the donor atoms get ionised and hence electron concentration in conduction band increases with temperature until all the donor atoms are ionised.

* This range is known as impurity or extrinsic range.



* When the temperature is further increased to room temperature, there are no more donor atoms to be ionised and hence the concentration of electrons in conduction band remains constant over a certain temperature range.

* This region is known as exhaustion range.

* The material practically becomes intrinsic and so this range is called intrinsic range.

* the dotted curve indicates hole concentration in an intrinsic range

---

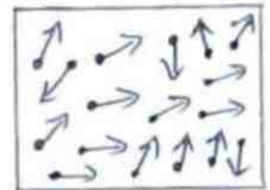
5. Carrier Transport in semiconductor Mobility:-

* In absence of an electrical field, the free electrons move in all directions in a random manner.

* They collide with other free electrons and positive ion core during the motion.

* This collision is known as elastic collision.

* As the motion is random, the resultant velocity in any particular direction is zero.



* The drift velocity attained by the carriers is proportional to the electrical field strength E

$$V_d \propto E$$

$$\boxed{V_d = \mu E} \quad \ldots (1)$$

Where μ is a proportionality constant and it is known as the mobility of charge carriers.

Mobility:-

If $E = 1V/m$ then $\mu = V_d$ Thus, mobility μ is defined as the velocity of a charge carrier per unit electrical field strength.

μ_n and μ_p denotes electron mobility and hole mobility respectively.

The values of electron and hole mobilities of 300K

Material	Electron mobility $m^2/volt-sec$	Hole mobility $m^2/volt-sec$
Silicon	0.135	0.048
Germanium	0.39	0.19

Expression for Electrical conductivity:-

$$V_{dn} = \mu_n E \quad \ldots (2)$$

where μ_n is the mobility of electron.

The drift current density J_n.

$$J_n = ne v_{dn} \quad \ldots (3)$$

$$J_n = \sigma_n E \quad \ldots (4)$$

$$\sigma_n = \frac{J_n}{E} = \frac{ne v_{dn}}{E} \quad \ldots (5)$$

Substituting eqn (2) in eqn (5),

$$\sigma_n = \frac{ne\mu_n E}{E}$$

$$\boxed{\sigma_n = ne\mu_n} \quad \ldots (6)$$

the drift of holes, then 'σ_p' the conductivity due to

$$\boxed{\sigma_p = pe\mu_p} \quad \ldots (7)$$

μ_p is the mobility of holes in the material

Total conductivity σ due to free electrons & holes

$$\sigma = \sigma_n + \sigma_p$$

$$\sigma = ne\mu_n + pe\mu_p$$

$$\sigma = e(n\mu_n + p\mu_p) \quad \ldots (8)$$

$$n = p = n_i$$

from eqn (8)

$$\sigma_i = e(n_i \mu_n + n_i \mu_p)$$

$$\boxed{\sigma_i = e n_i (\mu_n + \mu_p)} \quad \ldots (9)$$

6. Drift and Diffusion:-

The net current flows across a semiconductor has two components.

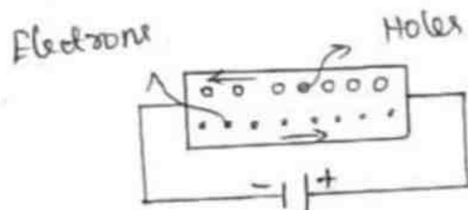
(i) Drift current

ii) Diffusion current

Drift current:-

* The charge carriers are forced to move in a particular direction due to the electric field.

* This is known as the drift motion and the current is known as drift current.



Drift current density, in a semiconductor due to electrons

$$\boxed{J_n = n \mu_n e E} \quad \ldots (1)$$

Drift current density due to hole,

$$\boxed{J_p = p \mu_p e E} \quad \ldots (2)$$

Total drift current density.

$$\boxed{J = J_n + J_p}$$

$$\boxed{J = ne\mu_n E + pe\mu_p E} \quad \ldots (3)$$

for intrinsic semiconductor. $(\because n = p = n_i)$

$$\boxed{J = n_i e (\mu_n + \mu_p) E}$$

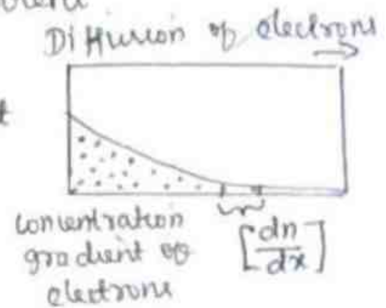
(8)

Diffusion current

Definition:

* The charge carriers move from the regions of higher concentration to the regions of lower concentration.

* This process is known as diffusion. The current is known as diffusion current.



negative sign denotes through denotes that the electrons are diffusing from higher concentration to lower concentration region.

$$\left.\begin{array}{l}\text{Rate of flow of electrons through} \\ \text{unit area}\end{array}\right\} = -D_n\left(\frac{dn}{dx}\right)$$

D_n is a proportionality constant and it is known as diffusion coefficient of electrons.

$$= -e \times -D_n\left(\frac{dn}{dx}\right)$$

$$\boxed{J_n = eD_n\left(\frac{dn}{dx}\right)}$$

Similarly, the diffusion current density of holes is given by,

$$\boxed{J_p = -eD_p\left(\frac{dp}{dx}\right)}$$

D_p is diffusion constant of holes.

7. Hall Effect:

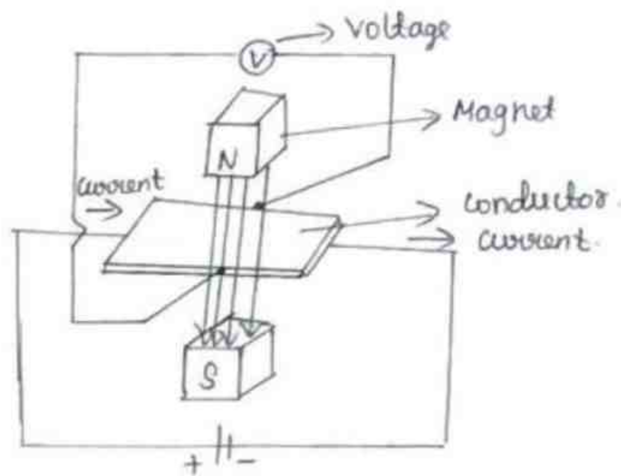
* The electrical conductivity measurements are not sufficient for the determination of number of charge carriers and their mobilities.

* Moreover, these measurements do not indicate whether current conduction is due to electrons or holes

Statement :-

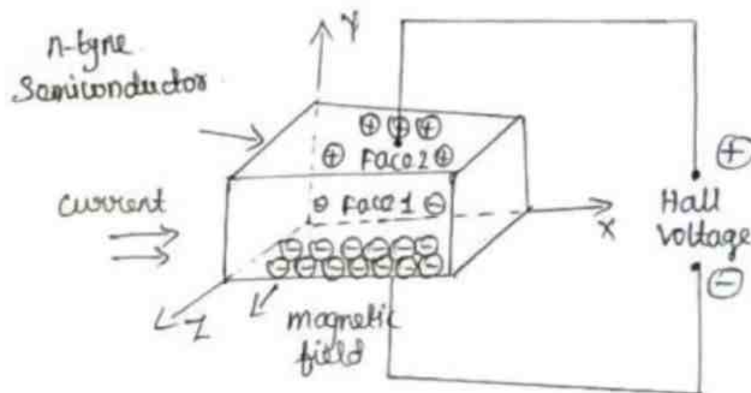
When a conductor carrying a current (I) is placed perpendicular to a magnetic field (B), a potential difference is produced inside the conductor in a direction perpendicular to both current and magnetic field.

This phenomenon is known as Hall effect. The voltage thus generated is called Hall voltage.



Hall effect in n-type semiconductor :-

Consider a n-type semiconductor in the form of a rectangular slab. In this slab, the current flows in x-direction and magnetic field B is applied in z-direction



(10)

upward force acting on each electron = eE_H — (2)

At equilibrium, downward force balance upward force.

$$\therefore Bev = eE_H$$

$$E_H = Bv \longrightarrow (3)$$

$$J_x = -nev \longrightarrow (4)$$

n is the concentration electrons.

$$v = \frac{-J_x}{ne} \longrightarrow (5)$$

Substituting eqn (5) in (3),

$$E_H = \frac{-BJ_x}{ne} \longrightarrow (6)$$

$$E_H = R_H J_x B \longrightarrow (7)$$

$$\left(\because R_H = -\frac{1}{ne}\right)$$

$$\boxed{R_H = \frac{E_H}{J_x B}} \longrightarrow (8)$$

R_H is a constant and it is known as Hall coefficient.

Hall effect in p-type semiconductor:-

Similar to n-type semiconductor, we can write for p-type semiconductor.

$$E_H = R_H J_x B$$

where Hall coefficient.

$$R_H = +\frac{1}{pe}$$

where p is concentration of holes.

Hall coefficient in terms of Hall voltage:

$$V_H = E_H t$$

where E_H is Hall field ... (1)

$$V_H = R_H J_x B t \quad \cdots (2)$$

(11)

Cross sectional area of the sample (A)

$$= \text{Breadth } (b) \times \text{Thickness } (t)$$
$$= bt$$

Current density $J_x = \dfrac{I_x}{\text{Area of the sample (A)}}$

$$= \frac{I_x}{bt} \quad \ldots \quad (3)$$

Substituting eqn (3) in eqn (2)

$$V_H = \frac{R_H I_x B t}{bt}$$

$$V_H = \frac{R_H I_x B}{b}$$

Hall coefficient $\boxed{R_H = \dfrac{V_H b}{I_x B}} \quad \ldots \quad (4)$

Applications of Hall effect:-

(i) Determination of semiconductor type:-

The sign of the Hall coefficient is used to find whether a given semiconductor is n-type or p-type.

ii) Calculation of carrier concentration:-

By measuring Hall coefficient R_H, carrier concentration is determined from the relation.

$$\boxed{n = \frac{1}{e R_H}}$$

iii) Determination of mobility:-

we know that electrical conductivity,

$$\sigma_e = ne\mu_e$$

$$\mu_e = \frac{\sigma_e}{ne} \qquad \left(\because \frac{1}{ne} = R_H\right)$$

$$\boxed{\mu_e = \sigma_e R_H}$$

8) Carrier concentration in intrinsic semiconductors:-

Definition:-

The number of electrons in conduction band per unit volume of the material is called as electron concentration (n)

The number of holes in valence band per unit volume of the material is called as hole concentration (p)

Density of Electrons in conduction Band :-

$$dn = Z(E)F(E)\,dE \quad \ldots (1)$$

$Z(E)\,dE$ -> Density of states in energy between E and E+dE.

$F(E)$ -> probability of electron occupancy.

Integrating eqn (1),

$$\int dn = n = \int_{E_c}^{\infty} Z(E)\,dE\,(F(E)) \quad \ldots (2)$$

Thus, in eqn (3), E is replaced as $(E-E_c)$

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E)^{1/2}\,dE \quad \ldots (3)$$

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E-E_c)^{1/2}\,dE \quad \ldots (4)$$

Fermi distribution function,

$$F(E) = \frac{1}{1+e^{(E-E_F)/k_BT}} \quad \ldots (5)$$

Substituting eqn (4) and (5) in (2)

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3}(2m_e^*)^{3/2}(E-E_c)^{1/2} \times \frac{1}{1+e^{(E-E_F)/k_BT}}\,dE \quad \ldots (6)$$

E, E_c, E_F, E_V, $+\infty$, $-\infty$

$KT \gg (E-E_F)$

Hence, '1' from the denominator of eqn (6) is neglected.

$$1 + e^{(E-E_F)/kT} = e^{(E-E_F)/kT}$$

eqn (6) becomes,

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2} e^{-(E-E_F)/kT}\, dE$$

$$= \frac{4\pi}{h^3}(2m_e^*)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2} e^{(E_F-E)/kT}\, dE$$

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{E_F/kT}\int_{E_c}^{\infty}(E-E_c)^{1/2} e^{-E/kT}\, dE \quad \ldots (7)$$

To evaluate above integral in eqn (7).

$E - E_c = x$	$E = E_c$	$E = +\infty$
$E = E_c + x$	$E_c - E_c = x$	$+\infty - E_c = x$
$\therefore dE = dx$	$\therefore x = 0$	$\therefore x = +\infty$

Substituting above values in eqn (7).

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{E_F/kT}\int_{E_0}^{\infty}(x)^{1/2} e^{-(E_c+x)/kT}\, dx$$

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F-E_c)/kT}\int_{0}^{\infty}(x)^{1/2} e^{-x/kT}\, dx \quad \ldots (8)$$

Using the gamma function,

$$\int_{0}^{\infty}(x)^{1/2} e^{-x/kT}\, dx = \frac{(kT)^{3/2}\pi^{1/2}}{2} \quad \ldots (9)$$

Substituting eqn (9) in eqn (8),

$$n = \frac{4\pi}{h^3}(2m_e^*)^{3/2} e^{(E_F-E_c)/kT}\left[\frac{(kT)^{3/2}\pi^{1/2}}{2}\right]$$

$$n = \frac{2\pi}{h^3}(2m_e^*)^{3/2}(kT)^{3/2}\pi^{1/2} e^{(E_F-E_c)/kT}$$

(14)

$$n = \frac{2\pi\pi^{1/2} (2m_e^*)^{3/2} (kT)^{3/2} e^{(E_F - E_c)/kT}}{(h^2)^{3/2}}$$

$$\boxed{n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_F - E_c)/kT}} \quad \ldots (10)$$

Eqn (10) is the expression for concentration of electrons in the conduction band of intrinsic semiconductor.

---

9) Density of holes in valence band of intrinsic semiconductor

$$dP = Z(E)(1 - F(E))\,dE \quad \ldots (1)$$

$Z(E)\,dE \rightarrow$ density of states in the energy range E and $E + dE$

$$1 - F(E) = 1 - \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$= \frac{1 + e^{(E-E_F)/kT} - 1}{1 + e^{(E-E_F)/kT}}$$

$$1 - F(E) = \frac{e^{(E-E_F)/kT}}{1 + e^{(E-E_F)/kT}} \quad \ldots (2)$$

$e^{(E-E_F)/kT}$ is very small and it is neglected in the denominator term of eqn (2).

$$1 + e^{(E-E_F)/kT} = 1$$

$$1 - F(E) = e^{(E-E_F)/kT} \quad \ldots (3)$$

Density of states in valence band.

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2} E^{1/2}\,dE \quad \ldots (4)$$

The term E in eqn (4) is replaced as $(E_v - E)$.

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2}\,dE \quad \ldots (5)$$

eqn substituting eqns (3) & (5) in (1).

$$dP = \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2} e^{(E-E_F)/kT}\,dE \quad \ldots (6)$$

integrating eqn (6)

$$\int dP = P = \int_{-\infty}^{E_V} \frac{4\pi}{h^3}(2m_h^*)^{3/2}(E_V - E)^{1/2} e^{(E-E_F)/kT}\,dE$$

$$P = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{-E_F/kT}\int_{-\infty}^{E_V}(E_V - E)^{1/2} e^{E/kT}\,dE \quad \ldots (7)$$

To evaluate the integral in eqn (7).

$E_V - E = x$	$E = -\infty$	$E = E_V$
$E = E_V - x$	$E_V - (-\infty) = x$	$E_V - E_V = x$
$dE = (-dx)$	$E_V + \infty = x$	$x = 0$
	$x = \infty$	

Substituting these values in eqn (7).

$$P = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{-E_F/kT}\int_{\infty}^{0} x^{1/2} e^{(E_V - x)/kT}(-dx) \quad \ldots (8)$$

(-ve sign is omitted by interchanging the limits)

$$P = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{(E_V-E_F)/kT}\int_0^{\infty} x^{1/2} e^{(-x)/kT}\,dx \quad \ldots (9)$$

using the gamma function,

$$\int_0^{\infty} x^{1/2} e^{(-x)/kT} = \frac{(kT)^{3/2}\pi^{1/2}}{2} \quad \ldots (10)$$

$$P = \frac{4\pi}{h^3}(2m_h^*)^{3/2} e^{(E_V-E_F)/kT}\left[\frac{(kT)^{3/2}\pi^{1/2}}{2}\right]$$

$$P = \frac{2\pi}{h^3}(2m_h^*)^{3/2}(KT)^{3/2}\pi^{1/2}e^{(E_V-E_F)/kT}$$

$$P = 2\pi\pi^{1/2}\frac{(2m_h^*)^{3/2}(KT)^{3/2}e^{(E_V-E_F)/kT}}{(h^2)^{3/2}}$$

$$\boxed{P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}e^{(E_V-E_F)/kT}} \quad \ldots (11)$$

The eqn (11) is the expression for the concentration of holes in valence band of intrinsic semiconductor.

10) Intrinsic carrier concentration:-

In an intrinsic semiconductor, the number of electrons in conduction band is equal to the number of holes in valence band

$$n_i = n = p \quad \ldots (1)$$

$$n_i \times n_i = n_i^2 = np \quad \ldots (2)$$

Substituting the expressions of n and p in eqn (2)

$$n_i^2 = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}e^{(E_F-E_C)/kT} \times 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}e^{(E_V-E_F)/kT}$$

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3(m_e^* m_h^*)^{3/2}e^{(E_V-E_C)/kT}$$

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3(m_e^* m_h^*)^{3/2}e^{-E_g/kT} \quad \ldots (3)$$

Where $E_C - E_V = E_g$ is forbidden energy gap.

Taking square root on both sides in eqn (3)

$$(n_i^2)^{1/2} = \left(4\left(\frac{2\pi kT}{h^2}\right)^3(m_e^* m_h^*)^{3/2}e^{-E_g/kT}\right)^{1/2}$$

(17)

$$n_i = 4^{1/2} \left(\frac{2\pi kT}{h^2}\right)^{3/2} \left((m_e^* m_h^*)^{3/2}\right)^{1/2} \left(e^{-E_g/kT}\right)^{1/2}$$

$$\boxed{n_i = 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT}} \quad \ldots (4)$$

The eqn (4) is expression for intrinsic carrier concentration.

Limitations of intrinsic semiconductor:-

* Intrinsic semiconductors cannot be directly used to fabricate devices due to the following limitations.
* Electrical conductivity is low. Germanium bar has a conductivity of $1.67\ \Omega^{-1}m^{-1}$.
* Which is nearly 10^7 times smaller than that of copper

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ii) **Extrinsic OR impure semiconductors:-**

In a semiconductors material, if the charge carriers originate from impurity atoms. Which are doped to the original material, then this type of semiconductor is known as extrinsic or impure semiconductor.

Doping:- The addition of impurities to a pure semiconductor is known as doping and added impurity is called as doping agent or dopant.

Advantages of Extrinsic semiconductors:-

* Electrical conductivity is high
* Electrical conductivity can be altered to any desired value by controlling of doping concentration.
* Electrical conductivity is not a function of temperature

Types of Extrinsic semiconductors:-

The extrinsic semiconductors are classified into two types based on the type of impurity added.

(i) n - type semiconductor
ii) p - type semiconductor.

n- type semiconductor:-

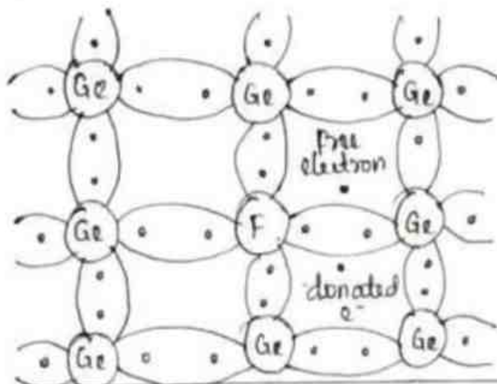
when a small amount of pentavalent impurity is doped to a pure semiconductor. it becomes n-type semiconductor.

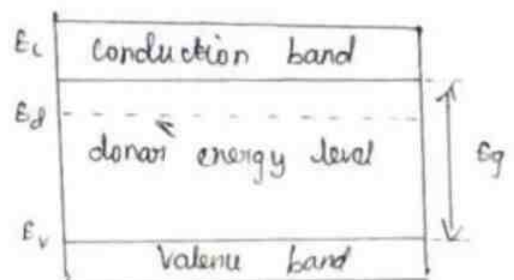
covalent bond in n-type semiconductor:

The addition of pentavalent impurity gives a large number of free electrons in semiconductor. Therefore it is called n- type semiconductor where n stands for negative type

Energy band of n- type semiconductor:-

when the donor impurities are added, the allowable energy levels are introduced





12) Carrier concentration in n-Type semiconductors :-

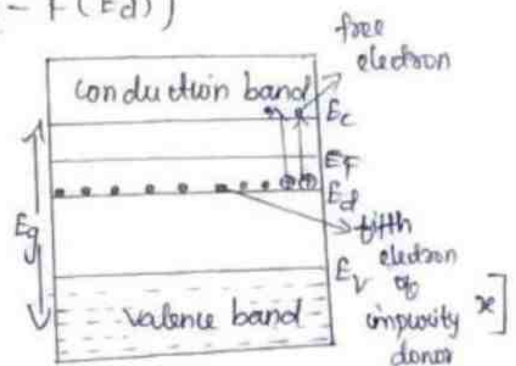
Density of electrons per unit volume in conduction band is given by

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_F - E_c)/kT} \quad \ldots (1)$$

$E_c \rightarrow$ Energy corresponding to the bottom most level of conduction band.

Density of ionized donors $= N_d(1 - F(E_d))$

The number of donor atoms per unit volume of the material.



$$= N_d\left[1 - \frac{1}{1+e^{(E_d - E_F)/kT}}\right] \quad \ldots (2)$$

$$= N_d\left[\frac{1 + e^{(E_d - E_F)/kT} - 1}{1+e^{(E_d - E_F)/kT}}\right]$$

$$= \frac{N_d\, e^{(E_d - E_F)/kT}}{1+e^{(E_d - E_F)/kT}} \quad \ldots (3)$$

$e^{(E_d - E_F)/kT}$ is very small in eqn (3), when compared to '1' it is neglected, $\therefore 1 + e^{(E_d - E_F)/kT} = 1$

Density of ionized donor $= N_d\, e^{(E_d - E_F)/kT} \quad \ldots (4)$

Eqn (1) and (4) equating,

$$2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_F - E_c)/kT} = N_d\, e^{(E_d - E_F)/kT} \quad \ldots (5)$$

rearranging the terms,

$$\frac{e^{(E_F - E_c)/kT}}{e^{(E_d - E_F)/kT}} = \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_F - E_c)/kT}\, e^{-(E_d - E_F)/kT} = \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_F - E_c - E_d + E_F)/kT} = \frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}} \quad \ldots (6)$$

Taking Log on both sides,

$$\log_e e^{(E_F - E_c - E_d + E_F)/kT} = \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

$$\frac{E_F - E_c - E_d + E_F}{kT} = \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right] \quad \left[\because \log_e e^x = x\right]$$

$$2E_F - (E_c + E_d) = kT \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

(or) $$2E_F = E_d + E_c + kT \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]$$

$$\boxed{E_F = \frac{E_d + E_c}{2} + \frac{kT}{2} \log_e \left[\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right]} \quad \ldots (7)$$

Substituting the expression of E_F from (7) in (1).

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp \left[\frac{\frac{(E_d + E_c)}{2} + \frac{kT}{2} \log_e \left\{\frac{N_d}{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}\right\} - E_c}{kT}\right] \quad \ldots (8)$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left[\frac{E_d + E_c - 2E_c}{2kT} + \frac{1}{2}\log_e\left\{\frac{N_d}{2\left\{\frac{2\pi m_e^* kT}{h^2}\right\}^{3/2}}\right\}\right]$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \exp\left[\frac{E_d - E_c}{2kT} + \log_e\left\{\frac{N_d^{1/2}}{\left\{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right\}^{1/2}}\right\}\right]$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{E_d - E_c/2kT}\, e^{\log_e\left\{\frac{N_d^{1/2}}{\left\{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\right\}^{1/2}}\right\}}$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{(E_d - E_c)/2kT} \frac{N_d^{1/2}}{2^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4}} \quad \ldots (9)$$

Rearranging the expression (9),

$$n = 2\,\frac{N_d^{1/2}}{2^{1/2}}\,\frac{\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}{\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4}}\, e^{(E_d - E_c)/2kT}$$

$$n = 2^{1/2} \times 2^{1/2}\,\frac{N_d^{1/2}}{2^{1/2}}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}\left(\frac{2\pi m^* kT}{h^2}\right)^{-3/4} e^{(E_d - E_c)/2kT}$$

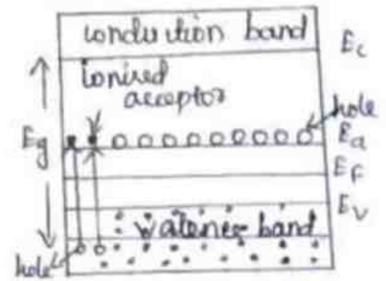
$$n = 2^{1/2} N_d^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} e^{(E_d - E_c)/2kT} \quad \ldots (10)$$

$$\boxed{n = (2N_d)^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} e^{-\Delta E/2kT}} \quad \ldots (11)$$

$\Delta E = E_c - E_d$ is the ionisation energy of the donor.

(3) Concentration of Holes in valence band of p-Type semiconductors :-

In p-type semiconductor, acceptor energy level is just above valence band.



Density of holes per unit volume in valence band is given by.

$$p = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_v - E_F)/kT} \quad \dots (1)$$

$E_v \rightarrow$ Energy corresponding to top most level of valence band

Density of ionized acceptors $= N_a F(E_a) \quad \dots (1)$

N_a - the number of acceptor atoms per unit volume.

$$F(E_a) = \frac{1}{1+e^{(E_a - E_F)/kT}}$$

E_a - acceptor energy level.

The eqn (1) becomes, density of ionized acceptors.

$$= \frac{N_a}{1+e^{(E_a - E_F)/kT}} \quad \dots (2)$$

$1 \ll e^{(E_a - E_F)/kT}$, thus '1' from the denominator of R.H.S. of eqn (2) is neglected.

$$N_a F(E_a) = \frac{N_a}{e^{(E_a - E_F)/kT}}$$

$$N_a F(E_a) = N_a e^{-(E_a - E_F)/kT}$$

Density of ionized acceptors $= N_a e^{(E_F - E_a)/kT} \longrightarrow (3)$

At equilibrium

$$\begin{pmatrix}\text{Density of holes} \\ \text{in valence band}\end{pmatrix} = \begin{pmatrix}\text{Density of ionized} \\ \text{acceptors}\end{pmatrix}$$

$$2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_v - E_F)/kT} = N_a e^{(E_F - E_a)/kT} \quad \dots\dots \rightarrow (4)$$

rearranging eqn (4),

$$\frac{e^{(E_v - E_F)/kT}}{e^{(E_F - E_a)/kT}} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_v - E_F)/kT} \cdot e^{-(E_F - E_a)/kT} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}$$

$$e^{(E_v - E_F - E_F + E_a)/kT} = \frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}} \quad \ldots\ldots (5)$$

Taking log on both sides in eqn (5), we have,

$$\log_e e^{(E_v - E_F - E_F + E_a)/kT} = \log_e\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right] \quad [\log_e e^x = x]$$

$$\frac{E_v - 2E_F + E_a}{kT} = \log_e\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]$$

$$E_a + E_v - 2E_F = kT\log_e\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right] \quad \ldots\ldots (6)$$

Rearranging.

$$2E_F = E_a + E_v - kT\log_e\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]$$

$$\boxed{E_F = \frac{E_a + E_v}{2} - \frac{kT}{2}\log_e\left[\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]} \quad \ldots\ldots (7)$$

Substituting eqn of E_F from (7) in (1)

$$P = 2\left(\frac{2\pi m_h^* kT}{2h^2}\right)^{3/2} \exp\left[\frac{E_V - \left[\left(\frac{E_a+E_V}{2}\right) - \frac{kT}{2}\log_e\left\{\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right\}\right]}{kT}\right]$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp\left[\frac{E_V - \left(\frac{E_V - E_a}{2}\right) - \frac{kT}{2}\log_e\left\{\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right\}}{kT}\right] \quad \ldots (8)$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp\left[\frac{2E_V - E_V - E_a}{2kT} + \frac{1}{2}\log_e\left\{\frac{N_a}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right\}\right]$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \exp\left[\frac{E_V - E_a}{2kT} + \log_e\left[\frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}\right)^{1/2}}\right]\right]$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_V - E_a)/2kT}\, e^{\log_e\left[\frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}}\right]}$$

$$P = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} e^{(E_V - E_a)/2kT} \quad \frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}}$$

$$P = 2\ \frac{N_a^{1/2}}{2^{1/2}}\ \frac{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}{\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}}\ e^{(E_V - E_a)/2kT}$$

$$P = \frac{2^{1/5} 2^{1/2} N_a^{1/2}}{2^{1/2}} \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2} \left(\frac{2\pi m_h^* kT}{h^2}\right)^{-3/4} e^{(E_v - E_a)/2kT}$$

$$P = (2N_a)^{1/2} \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} e^{(E_v - E_a)/2kT} \quad \ldots (9)$$

$$\boxed{P = (2N_a)^{1/2} \left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} e^{-\Delta E/2kT}} \quad \ldots (10)$$

where $\Delta E = E_a - E_v$ is the ionisation energy of acceptors.

14) Hall Devices:-

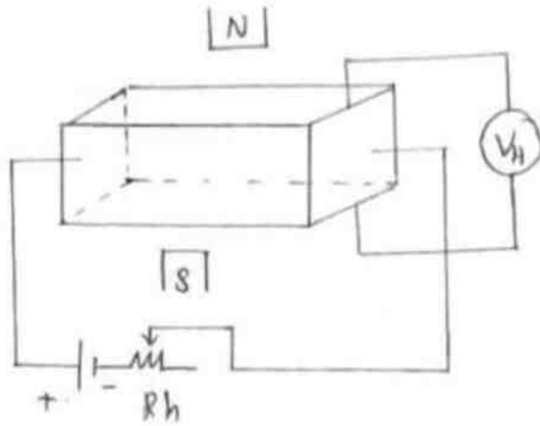
The device which uses the hall effect for its application is known as hall device.

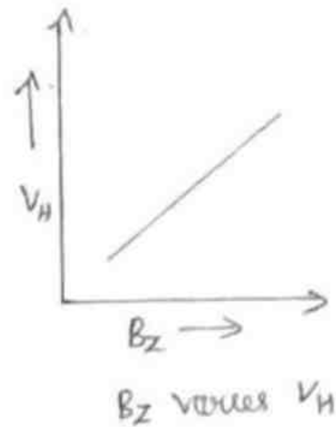
There are three types of Hall devices

* Gauss Meter
* Electronic Multiplier
* Electronic wattmeter

a) Gauss Meter:-

* The Hall voltage $V_H = \frac{R_H B_z I_x}{t}$. In this $V_H \propto B_z$ for a given hall element, R_H and t are constant.
* The current I through Hall element is also kept constant
* This principle is used in Gauss meter. It is used for measuring magnetic field.
* The graph can be also used to measure any unknown magnetic fields.





b) Electronic Multipliers:-

From Hall effect, $V_H = \dfrac{R_H B_Z I_1}{t}$

R_H and t are constant for an element.

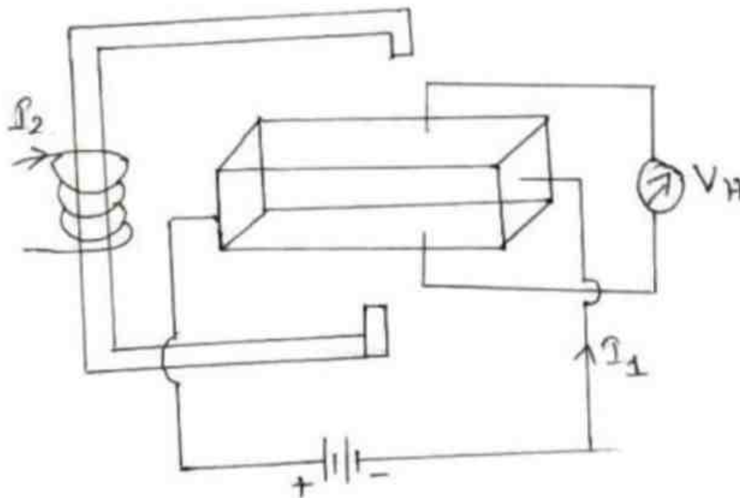
$V_H \propto B_Z I_1$

The magnetic field B_Z is proportional to current through the coil.

$B_Z \propto I_2$

$\therefore V_H \propto I_1 I_2$

V_H is a measure of the product of two currents. This is the basic principle used in analog electronic Multipliers.



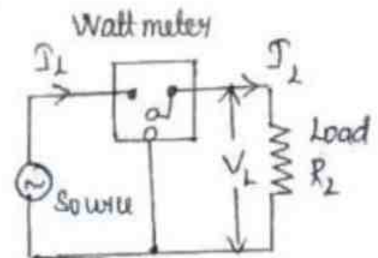
c) Electronic Wattmeter :-

* Hall effect is used to measure electrical power dissipated in a load.

* The instrument used to measure the power in a circuit using Hall effect principle is known as Hall effect Watt meter

* S is Hall effect sample. It is placed in a magnetic field B_z produced by the load current I_L passing through the coils CC.

If 't' thickness of the sample, then the measured Hall voltage



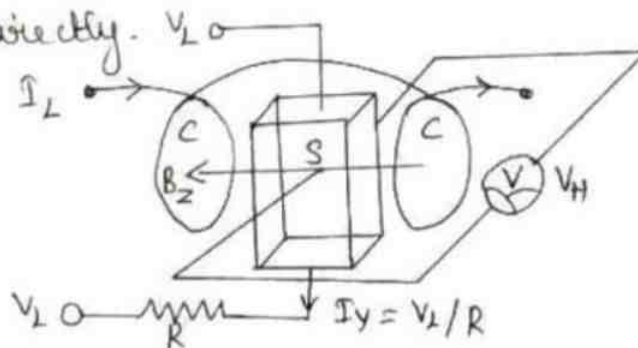
$$V_H = \frac{R_H B_Z I_Y}{t}$$

$$V_H \propto B_Z \cdot I_y$$

$$B_Z \propto I_L \text{ and } I_y \propto V_L$$

$$V_H \propto I_L V_L$$

This is the electric power dissipated by the load. The voltmeter that measures V_H can be calibrated to read power directly.



Metal - semiconductor (MS) constant :-

When a metal and a semiconductor are brought into contact, there are two types of junctions formed depending on the work functions of the metal and semiconductor.

(28)

Types of metal - semiconductor junction:-

i) schottky junction - $\phi_m > \phi_{semi}$

ii) ohmic junction - $\phi_{semi} < \phi_{semi}$

ϕ_m - work function of metal

ϕ_z - work function of semiconductor.

work function:- Energy required to raise the electrons from the metal or the semiconductor to the vacuum level.

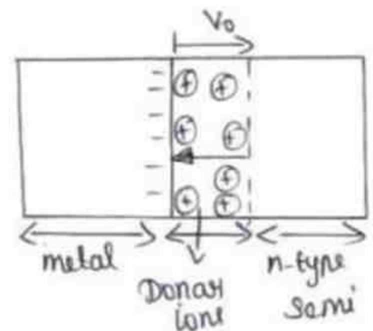
15) Schottky Diode:-

Definition:-

It is a junction formed between a metal and n-type semiconductor.

When the metal has a higher work function than that of n-type semiconductor then the junction formed is called schottky diode.

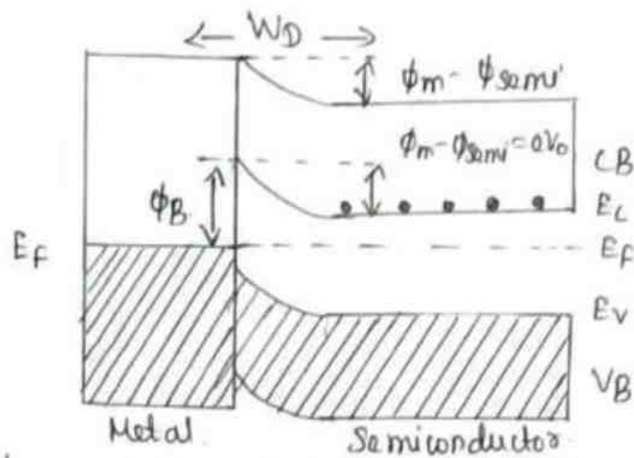
This leaves a positive charge on the semiconductor side and a negative charge on the metal side. This leads to a contact potential



Energy band diagram:-

When a shottky junction is formed between metal and semiconductor. fermi level lines up. Also a positive potential is formed on the semiconductor side.

$$eV_0 = \phi_m - \phi_{semi}$$



The contact potential thus formed prevents further motion of the electrons between the metal and semiconductor.

This is called the schottky barrier and denoted by ϕ_B.

Working:-

The behaviour of the schottky diode is further studied by biasing. This voltage is applied in two ways.

* Forward bias
* Reverse bias

Forward bias:-

* In this bias, metal is connected to positive terminal and n-type semiconductor is connected to negative terminal of the battery.

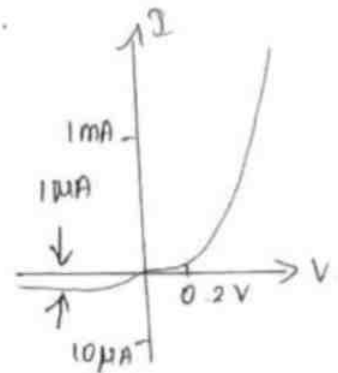
* This leads to a current in the circuit which increases with increasing external potential.

Reverse bias:-

* In reverse bias, metal is connected to negative terminal and n-type semiconductor to positive terminal of the battery.

so a schottky junction acts as a rectifier i.e. it conducts in forward bias but not in reverse bias.

V-I characteristics :-



The V-I characteristics of the junction, There is an exponential increase in current in the forward bias while there is a very small current in reverse bias.

Advantages of schottky diode :-

* In schottky diode, stored charges or depletion region is negligible. so a Schottky diode has a very low capacitance.
* It has high efficiency.
* It operates at high frequencies.
* It produces less noise.

Applications of schottky diode :-

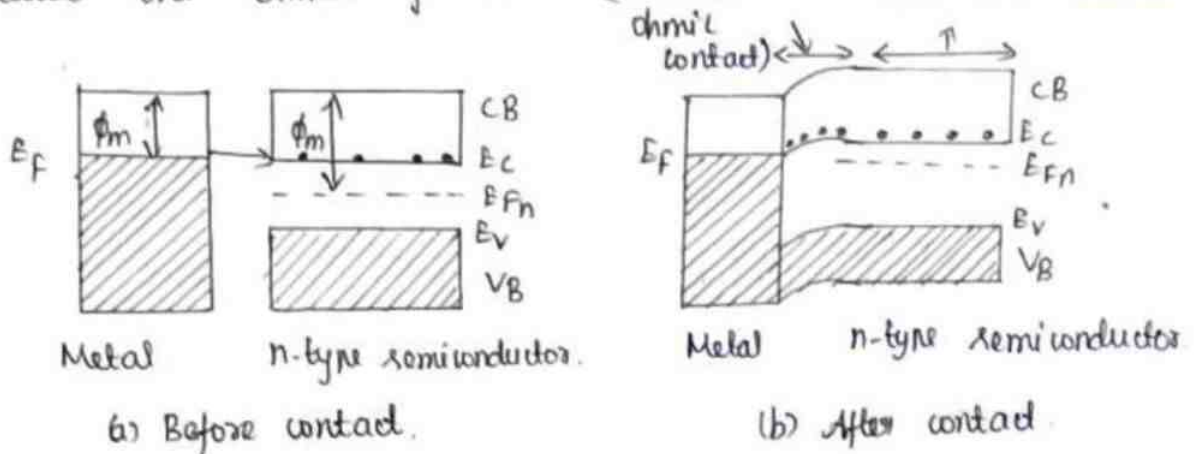
* Schottky diode can be used for rectification of signals of frequencies even exceeding 300 MHz.
* It is commonly used in Switching device at frequencies of 20 GHz.
* It is used in radio frequency applications
* It is widely used in power supplies.
* It is used to detect signals.
* It is used in logic circuits.
* Its low noise figure finds application in sensitive communication receivers like radars.

16) <u>Ohmic contacts:-</u>

<u>Definition:-</u>

An ohmic contact is a type of metal semiconductor junction. It is formed by a contact of a metal with a heavily doped semiconductor.

When the semiconductor has a higher work function than that of metal, then the junction formed is called the ohmic junction.



(a) Before contact.   (b) After contact

<u>Working:-</u>

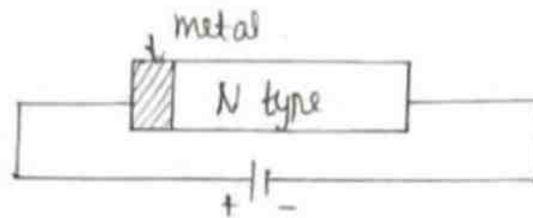
After contact the ohmic junction,

* At equilibrium, the electrons move from the metal to the empty states in the conduction band of semiconductor.

* Thus there is an accumulation region near the interface.

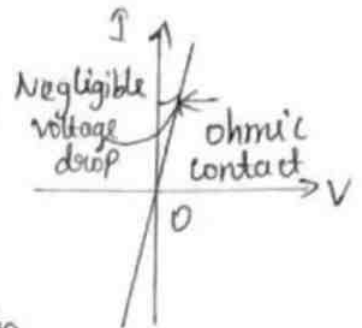
* Thus a ohmic contact behaves as a resistor conducting in both forward and reverse bias.

* The resistivity is determined by the bulk resistivity of the semiconductor.



V - I characteristics :-

The volt-ampere (V-I) characteristic of the ohmic contact,



The current is directly proportional to the potential across the junction and it is symmetric about the origin,

Thus, ohmic contacts are non-rectifying and show negligible voltage drop and resistance irrespective of the direction and magnitude of current.

Applications :-

The use of ohmic contacts is to connect one semiconductor device to another, an IC, or to connect an IC to its external terminals.

Unit - III

Problems:-

1. Find the resistance of an intrinsic germanium rod 1 cm long, 1 mm wide and 1 mm thick at 300 K, for germanium $n_i = 2.5 \times 10^{19} m^{-3}$, $\mu_e = 0.39 m^2 V^{-1} s^{-1}$, $\mu_h = 0.19 m^2 V^{-1} s^{-1}$ at 300K.

Given data:-

Intrinsic carrier concentration $n_i = 2.5 \times 10^{19} m^{-3}$

Electron mobility $\mu_e = 0.39 m^2 V^{-1} s^{-1}$

Hole mobility $\mu_h = 0.19 m^2 V^{-1} s^{-1}$.

L = length of the rod $= 1cm = 1 \times 10^{-2} m$.

A = Area of cross-section $(A = (1 \times 10^{-3})(1 \times 10^{-3}))$

Solution:-

Electrical conductivity of an intrinsic semiconductor.

$$\sigma = n_i e(\mu_e + \mu_h)$$

Substituting given values,

$$\sigma = 2.5 \times 10^{19} \times 1.6 \times 10^{-19} \times (0.39 + 0.19).$$

$$\sigma = 2.32 \Omega^{-1} m^{-1}$$

Resistance $R = \frac{\rho L}{A}$ or $R = \frac{L}{\sigma A}$ $\left(\because \sigma = \frac{1}{\rho}\right)$

$$R = \frac{1 \times 10^{-2}}{2.32 \times (1 \times 10^{-3} \times 1 \times 10^{-3})} = \frac{1}{2.32} \times 10^{-2} \times 10^{6}$$

$= 4310 \Omega$

Resistance of germanium $= 4310 \Omega$.

2. Find the concentration of holes and electrons in n-type silicon at 300 K, if the conductivity is $3 \times 10^{4} ohm^{-1} m^{-1}$. Also find these values for p-type silicon.

Given data:-

for silicon at 300K, $n_i = 1.5 \times 10^{16} m^{-3}$

$\mu_e = 1300 \times 10^{-4} m^2 V^{-1} s^{-1}$

$\mu_h = 500 \times 10^{-4} m^2 V^{-1} s^{-1}$

Solution:-

(a) concentration in n-type silicon:-

$\sigma = ne\mu_e$

$n = \frac{\sigma}{e\mu_e}$

$n = \frac{3 \times 10^4}{1.6 \times 10^{-19} \times 1300 \times 10^{-4}}$

$= 1.442 \times 10^{24} m^{-3}$

$np = n_i^2$

$p = \frac{n_i^2}{n} \Rightarrow p = \frac{(1.5 \times 10^{16})^2}{1.442 \times 10^{24}}$

$p = 1.56 \times 10^8 m^{-3}$

(b) concentration in p-type silicon:

$\sigma = pe\mu_h$

$p = \frac{\sigma}{e\mu_h}$

$p = \frac{3 \times 10^4}{1.6 \times 10^{-19} \times 500 \times 10^{-4}}$

$p = 3.75 \times 10^{24} m^{-3}$

$n = \frac{n_i^2}{p} = \frac{(1.5 \times 10^{16})^2}{3.75 \times 10^{24}}$

$n = 0.6 \times 10^8 m^{-3}$

2. A silicon material is uniformly doped with phosphorus atoms at a concentration of $2 \times 10^{19} m^{-3}$. The mobilities of holes and electrons are 0.05 and $0.12 m^2 V^{-1} s^{-1}$ respectively, $n_i = 1.5 \times 10^{16}$

find the electron and hole concentration and electrical conductivity.

Solution:-

Hole concentration $p = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{16})^2}{2 \times 10^{19}} = 1.125 \times 10^{13}/m^3$

Electron concentration $n = N_D = 2 \times 10^{19}/m^3$

Electrical conductivity $\sigma = e N_D \mu_e$

$\sigma = 1.602 \times 10^{-19} \times 2 \times 10^{19} \times 0.12$.

$\sigma = 0.384\ ohm^{-1} m^{-1}$.

4. find the hole and electron concentrations in a p-type semiconductor, if the acceptor density is 10^{20} atoms/m^3 and the intrinsic concentration is 2.5×10^{19} per m^3 at 300k.

Solution:-

$p = N_a = 10^{20}$ holes $/m^3$

$n = \frac{n_i^2}{N_a} = \frac{(2.5 \times 10^{19})^2}{10^{20}} = 6.25 \times 10^{18}$ electrons $/m^3$

$n = 6.25 \times 10^{18} m^{-3}$

5. The Hall coefficient of a specimen of a doped silicon is found to be $3.66 \times 10^{-4}\ m^3/c$. The resistivity of the specimen is $8.93 \times 10^{-3}\ \Omega m$. find the mobility and density of the charge carriers.

Given data:-

Hall coefficient of the specimen $R_H = 3.66 \times 10^{-4} m^3/c$.

Resistivity of the specimen $\rho = 8.93 \times 10^{-3}\ \Omega\, m$

Mobility of the carrier $\mu_h = ?$

Density of charge carrier $n_h = ?$

Solution :-

$$n_h = \frac{1}{R_H e}$$

substituting the given values, we have,

$$n_h = \frac{1}{3.66 \times 10^{-4} \times 1.610 \times 10^{-19}}$$

$$n_h = 1.708 \times 10^{22} \, m^{-3}$$

$$\mu_h = \frac{1}{\rho n_h e}$$

$$\mu_e = \frac{R_H}{\rho}$$

$$\mu_h = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}}$$

$$\mu_h = 0.041 \, m^2 V^{-1} s^{-1}$$

---

6. Find the Hall coefficient and electron mobility of germanium for a given sample. A current of 5 mA flows from a 1.35 Volt supply and develops a hall ~~val~~ voltage of 20 millivolt across the specimen in a magnetic field of 0.45 Wb/m².

Given data:-

current through the specimen $I = 5\,mA$ or $5 \times 10^{-3} A$.

voltage across the speciman $V = 1.35 V$

Length of the sample $L = 1\,cm$ or $1 \times 10^{-2} m$.

Breadth of the sample $b = 5\,mm$ or $5 \times 10^{-3} m$.

Thickness of the sample $t = 1\,mm$ or $1 \times 10^{-3} m$.

Hall voltage $V_y = 20 \times 10^{-3} V$

Magnetic field $H = 0.45 Wb/m^2$.

Solution :-

resistivity $\rho = \frac{Ra}{l}$

$R \rightarrow$ Resistance of the specimen,

$R = \frac{V}{I} = \frac{1.35}{5 \times 10^{-3}}$

$a \rightarrow$ Area of cross-section $= b \times t$.

$a = 5 \times 10^{-3} \times 1 \times 10^{-3}$

$= 5 \times 10^{-6} m^2$.

$\rho = \frac{1.35}{5 \times 10^{-3}} \times \frac{5 \times 10^{-6}}{1 \times 10^{-2}} = 0.135 \Omega m$.

Hall field $E_y = \frac{V_y}{\text{thickness}} = \frac{20 \times 10^{-3}}{1 \times 10^{-3}} = 20 V m^{-1}$

Current density $J_x = \frac{\text{current}}{\text{Area of cross-section}}$

$J_x = \frac{5 \times 10^{-3}}{5 \times 10^{-6}} = 1 \times 10^3 Am^{-2}$

$\frac{1}{ne} = \frac{E_y}{HJ_x} = \frac{20}{0.45 \times 10^3}$

$= 0.044 m^3/C$.

Hall coefficient $R_H = 1.18 \times \frac{1}{ne} = 1.18 \times 0.044$

$= 0.0524 m^3/C$

$R_H = 0.0524 m^3/C$.

Electron mobility $\mu_e = \frac{R_H}{\rho} = \frac{0.0524}{0.135}$

$= 0.39 m^2 V^{-1} S^{-1}$

$\mu_e = 0.39 m^2 V^{-1} s^{-1}$.

7. A copper strip 2.0 cm wide and 1.0 mm thick is placed in a magnetic field with B = 1.5 weber / m^2 perpendicular to the strip. suppose a current of 200 A is set up in the strip. what hall potential difference would appear across the strip? Given N = 8.4×10^{28} e/m^3.

Given data:-

Current flowing $I_x = 200A$.

Applied magnetic field $H_z = 1.5 Wb m^{-2}$

Numbers of electrons per unit volume $n = 8.4 \times 10^{28} e m^{-3}$

Thickness of the strip $t = 1.0 \times 10^{-3} m$

Solution:-

$$\text{Hall potential } V_y = \frac{R_H I_x B_z}{b}$$

$$V_y = \frac{I_x B_z}{neb} \qquad \left(\because R_H = \frac{1}{ne}\right)$$

Substituting the given values,

$$V_H = \frac{200 \times 1.5}{(8.4 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-3})}$$

$$V_H = 2.2 \times 10^{-5} V.$$

unit - IV

Optical properties of Materials

1. classification of optical Materials:-

Generally, optical materials are classified into three types based on the nature of propagation of light namely.

* Transparent
* Translucent
* opaque

(i) Transparent:-

* Transparent materials are the materials which transmit the light with little absorption and reflection.
* These materials are transparent in nature and hence, one can clearly view the object through the material.
* Electrical insulated materials are transparent. Similarly few semiconducting materials are also transparent.

ii) Translucent:-

* The incident light gets scattered within the materials and hence, the diffused light is transmitted with the other side of the material.
* one cannot clearly view the object while viewing through the materials.
* these materials are known as translucent material.

iii) opaque:-

* The material which absorbs the visible light is termed as opaque.
* when an electromagnetic radiation in the entire visible spectrum is incident on this material, either it gets reflected or absorbed.
* Thus, the materials are opaque. Few semiconducting materials also exhibit this opaque nature

2. optical processes in semiconductors:-

* When light incident on a semiconductor, an electron in the valence band goes to the conduction band.

* This process generates electron-hole pairs.

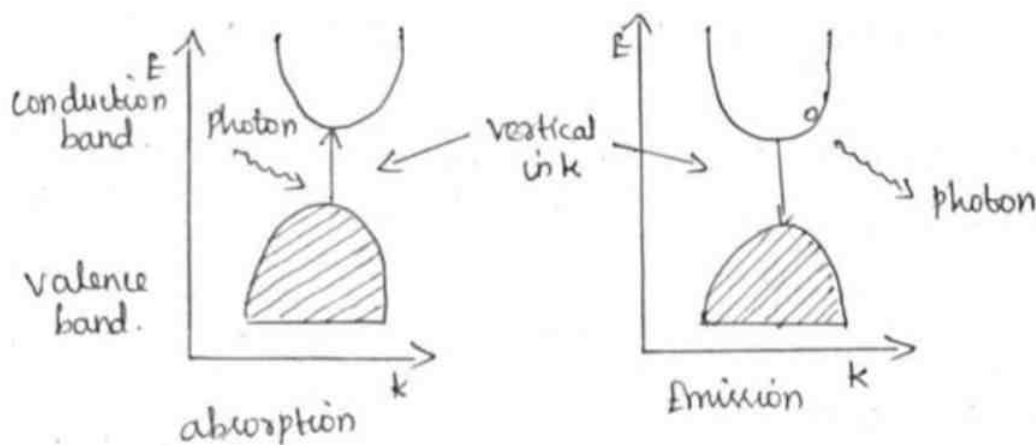
* It is also possible for an electron and a hole to recombine and emit light.

* The interaction between light and electrons in the semiconductor provides a variety of phenomena.

* These are used in the field of opto electronics.

* Thus electron jumps to the conduction band and leaving a hole in VB.

* In the reverse process the electron in the conduction band recombines with a hole in the valence band to generate a photon.



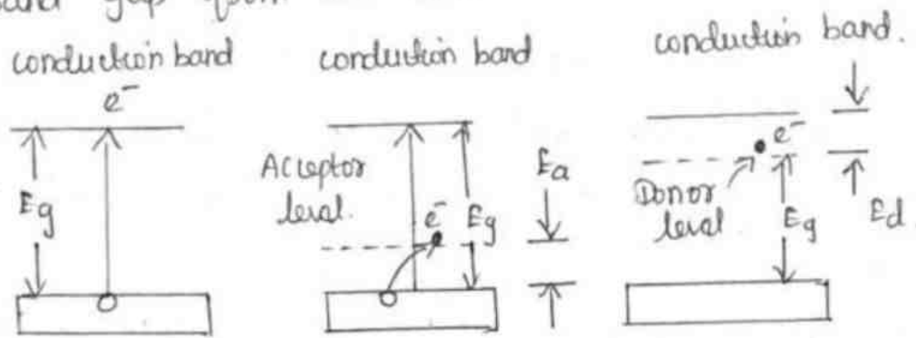
These two processes are importance for light-detection and light-emission devices.

3. Absorption and emission of light in semiconductors:.

* In semiconductors, light photons is absorbed in several ways.

* In intrinsic semiconductors such as Si, Ge and GaAs, light photons is absorbed to create electron-hole pairs.

This absorption causes electrons to jump across the energy band gap from the valence band to the conduction band.



$$\boxed{h\nu > E_g} \quad \ldots (1)$$

where h – planck's constant

ν – frequency of the light photon.

In terms of wavelength λ $\qquad \left[\because \nu = \frac{c}{\lambda}\right]$

$$\boxed{\frac{hc}{\lambda} > E_g} \quad \ldots (2)$$

The maximum wavelength for visible light λ_{max} is about 0.7 μm.

$$\boxed{E_{g(min)} = \frac{hc}{\lambda_{max}}} \quad \ldots (3)$$

Substituting the corresponding values, we have,

$$E_{g(min)} = \frac{(6.62 \times 10^{-34})(3 \times 10^{8})}{0.7 \times 10^{-6}}$$

$$= 2.84 \times 10^{-19} J$$

$$= \frac{2.84 \times 10^{-19}}{1.6 \times 10^{-19}} \qquad (\because 1 eV = 1.6 \times 10^{-19} J)$$

$$\boxed{E_{g(min)} = 1.8 eV} \quad \ldots (4)$$

The result indicates that all visible light is absorbed by those semiconductors having band gap energies less than about 1.8 eV. Thus, these semiconductors are opaque.

4. Charge injection and Radiative Recombination.

* Electrons and holes can be injected into the conduction and valence band in a number of way.

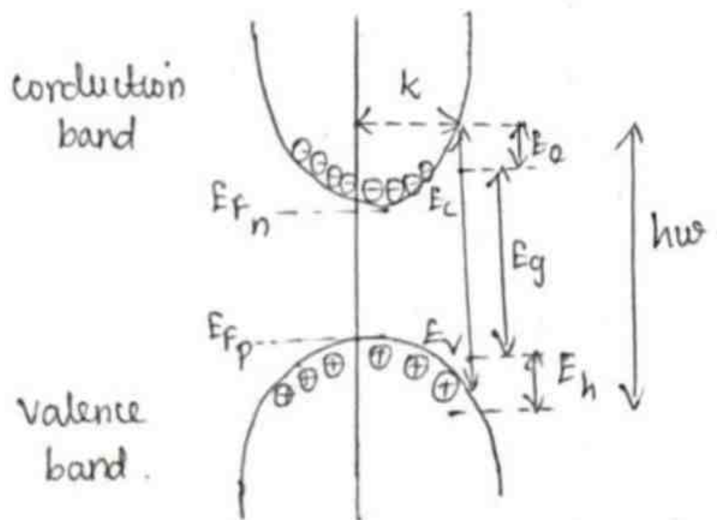
* This recombination process can be made in two processes. They are (i) radiative processes and (ii) non-radiative processes.

* In the radiative process the e-h pair recombines and a photon is emitted.

* This is the inverse of the photon absorption process.

* Electron-hole pairs can be also recombine without emitting light.

* Instead, they may emit (i) heat or (ii) a photon or (iii) a long-wavelength photon together with a phonon. such processes are non-radiative processes.



Types of carrier injections:-

(i) Minority carrier Injection:

If $n \gg p$ and the sample is heavily doped n-type recombination rate is proportional to hole density.

(ii) Strong injection:-

This case is important when a high density of both electrons and holes is injected.

iii) weak injection:-

In this case, the rate of recombination is very low.

iv) At low injection:-

The electrons have a low probability to find a hole with which to recombine.

---

5. Carrier Generation and Recombination processes:-

The carrier generation is the process whereby electrons and holes are created. The recombination is the process whereby electrons and holes are annihilated.

Carrier Generation:-

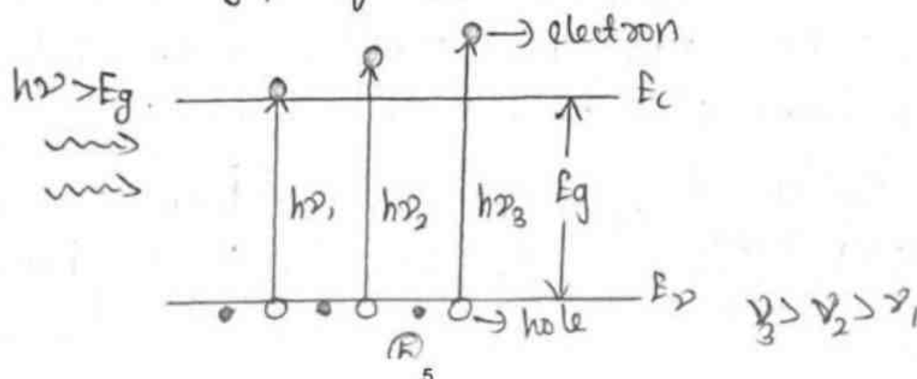
Basically, there are three types of carrier generation. They are

* Photogeneration
* phonon generation
* Impact ionization

(i) Photogeneration:-

* In photogeneration light of frequency ν falls on a semiconductor.

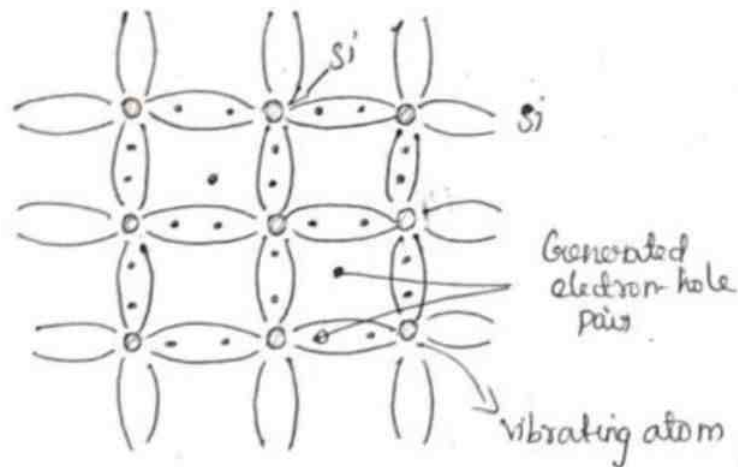
* Let $h\nu$ be the energy of light photon greater than the bandgap of the semiconductor.



(F)

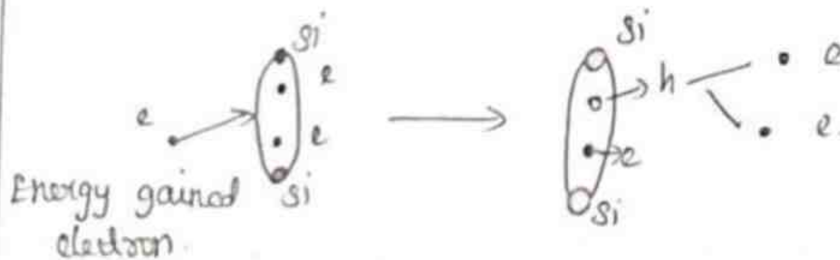
ii) Phonon Generation:-

Phonon generation occurs when a semiconductor is under thermal excitation, with increase in temperature of the semiconductor, lattice vibrations increase which give rise to more phonons.



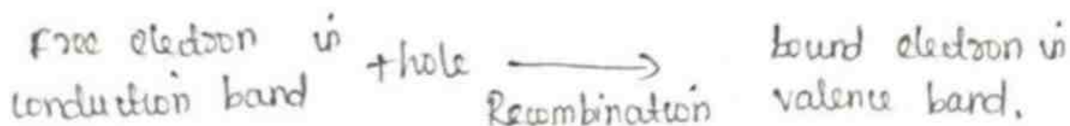
iii) impact ionization:-

In this process, a bond breaks out generating more carriers for a very high electric field, it results in a avalanche breakdown.



Recombination:-

In recombination, a pair of electron and hole gets recombined. when a free electron in the conduction band falls to valence band and recombines with a hole, it becomes a bound electron in valence band.

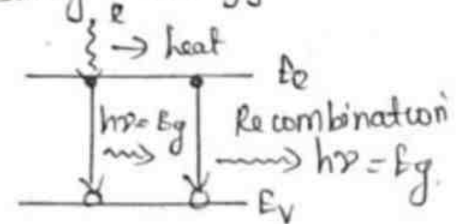


6. Recombination occurs in three ways:-

* Radiative Recombination
* Shockley - Read - Hall Recombination
* Auger Recombination.

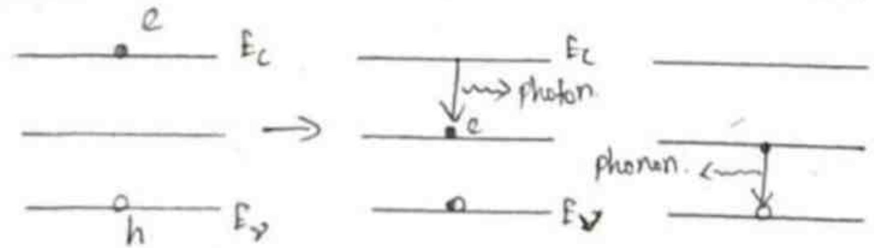
a) Radiative Recombination:-

Electrons which are excited to higher energy states in conduction band will come to the conduction band minimum by releasing energy as heat.



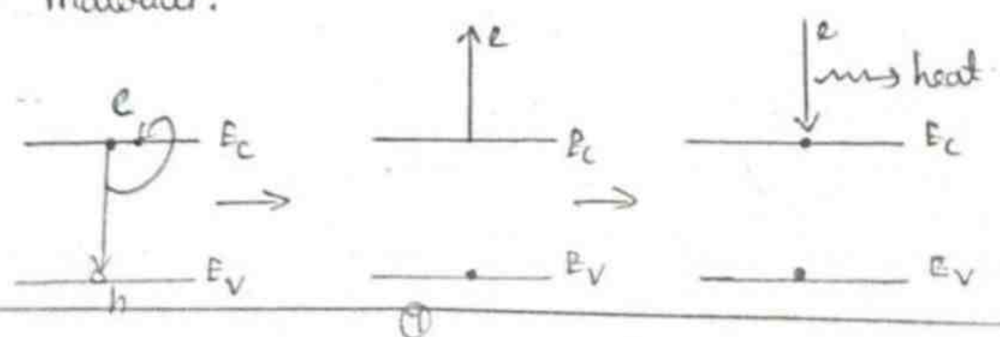
b) Shockley - Read - Hall Recombination:-

This type of recombination is basically seen in impure semiconductor which has defect levels. Generally, the defect level lies in the middle of the forbidden gap.



c) Auger Recombination:-

Then, the third excited electron comes back to the conduction band edge by emitting energy as heat. Generally, an Auger recombination occurs for heavily doped material.



7. <u>Optical Absorption, Loss And Gain:</u>

The photon flux associated with an electromagnetic wave traveling through a semiconductor is denoted by,

$$\boxed{I_{ph} = I_{ph}^{0} \exp(-\alpha x)} \quad \dots (1)$$

where α is usually positive and, I_{ph}^{0} is the incident light intensity at $x=0$.

The gain coefficient is defined by,

gain coefficient = emission coefficient - absorption coefficient.

For these transitions we have,

$$E^{e} = E_c + \frac{m_r^*}{m_e^*}(h\nu - E_g)$$

$$\boxed{E^{h} = E_v + \frac{m_r^*}{m_h^*}(h\nu - E_g)} \quad \dots (2)$$

The occupation probabilities f^e and f^h are found by the quasi-fermi levels for electrons and holes.

$$g(h\nu) \sim f^e(E^e) \cdot f^h(E^h) - \{1 - f^e(E^e)\}(1 - f^h(E^h))$$

$$\boxed{g(h\nu) = \{f^e(E^e) + f^h(E^h)\} - 1} \quad \dots (3)$$

The optical wave has a general spatial intensity dependence:

$$\boxed{I_{ph} = I_{ph}^{0} \exp(h\nu) x} \quad \dots (4)$$

The condition for positive gain requires "inversion" of the semiconductor system, eqn (3).

$$\boxed{f^e(E^e) + f^h(E^h) > 1} \quad \dots (5)$$

The quasi-fermi levels must penetrate their respective bands for this condition to be satisfied. (8)

8. optical processes in Quantum wells:-

* The term "Well" refers to a semiconductor region that is grown to possess a lower energy, so that it acts as a trap for electrons ~~f~~ and holes.

* These are called quantum wells because these semiconductor regions are only a few atomic layers thick.

* Quantum wells are real-world implementation of the "particle in one-dimensional box" ~~part~~ problem.

$$E_n = \frac{h^2 n^2}{8 m^* L^2} \qquad \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \ldots (1)$$

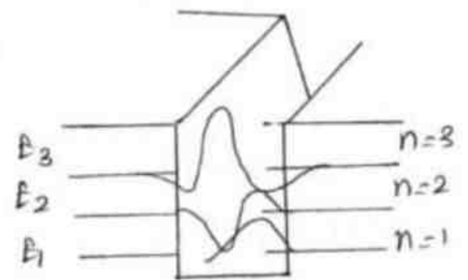
$n = 1, 2, \ldots \infty$ Quantum numbers.

h = planck's constant

m = mass of charge carrier.

energies and wave functions for a finite depth well.

The energy of the first allowed electron energy level in a typical 100 Å GaAs quantum well is about 40 meV calculated using eqn (1)

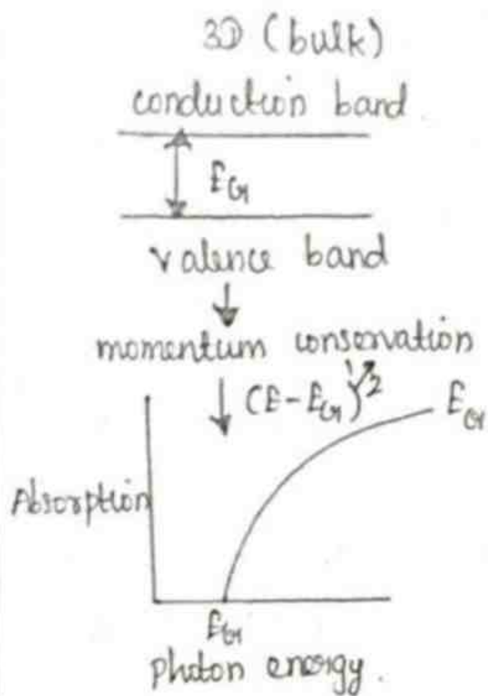


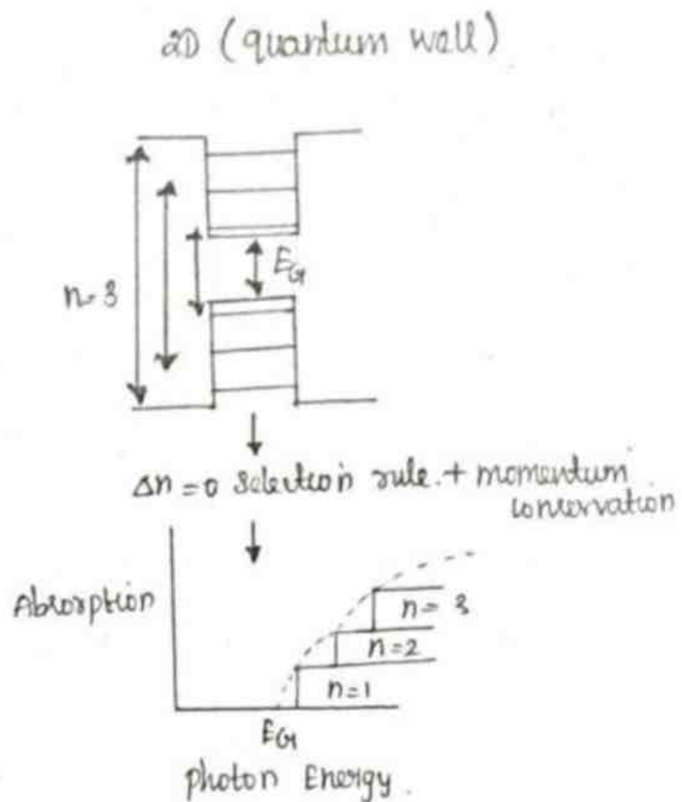
This rule follows from the fact that the optical absorption strength is proportional to the overlap integral of the conduction and valence wavefunctions.

In quantum well the electrons and holes are still free to move in the directions parallel to the layers.

Therefore, there is deviation in discrete energy states for electrons and holes.

⑨





$$E_g^{ef} = E_g + \frac{h^2}{8m_e^* L^2} + \frac{h^2}{8m_h^* L^2}$$

* This effective bandgap will determine the energy of the emitted photons.
* The bandgap can be altered by varying the thickness of the well.
* The carrier energy is quantized for the motion normal to the well but within the well motion is unrestricted.

---

9. Optoelectronic Devices:

* optoelectronic devices play a vital role in the fiber optics communication, switching and logic gates.
* The importance of optoelectronics applications results from the advances in semiconductor materials, optical fiber communication, optical data processing, display devices and data storage devices.

Light Detectors:-

Definition:-

It is a device which converts light signal into electrical wave forms.

Types of photo-detectors:-

photo-detectors are of three types:.

* photo emissive
* photo conductive
* photo voltaic

Photo-emissive photo-Detector:-

The emission of electrons from a photo cathode by the incident photon is called photo-emission.

Examples of such devices:-

* photo-tubes
* photo-multiplier tubes

The size of these is normally very large and hence not suitable for use as fibre optic detectors.

photo-conductive Devices:-

These types of devices have variation of resistance due to incident light on the photo-conductive materials.

Example of photo-conductive materials:-

* Materials like cds.
* Intrinsic semiconductor materials like PbS, PbTe.
* Extrinsic Semiconductor like doped Ge and Si
* They are not suitable for use in fibre optic communication purposes since they have low frequency response.

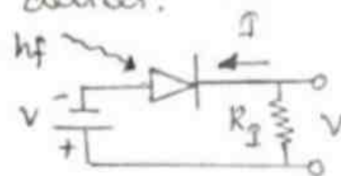
Photo-voltaic Devices:-

semiconductor junction photo diodes are called as photo-voltaic devices.

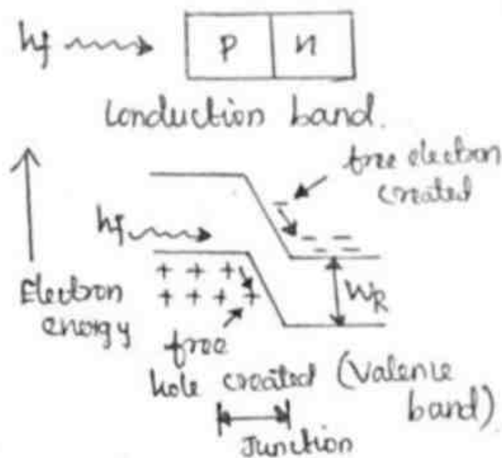
(11)

They are almost ideal for fibre systems.

We will study three forms of these devices.



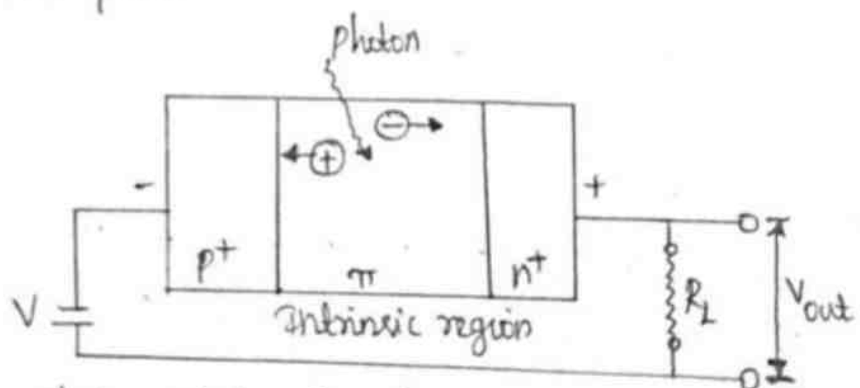
1. PN junction photo detector.
2. PIN photo diode
3. Avalanche photo diode (APD)



* Incident photon being absorbed in the junction after passing through the P-layer.
* The absorbed energy raises a bound electron across the bandgap.

PIN photo Diode:-

* The frequency response can be improved if the pn junction is separated by an intrinsic region.
* The introduction of the intrinsic region decreases the junction capacitance. This is called positive Intrinsic Negative (PIN) photo diode.
* The photons incident on the intrinsic region produce electron - hole pair.



Avalanche photo-Diodes (APD)

The working of Avalanche photo-diode. It is much more sensitive than PIN or PN diodes.

The avalanche photodiode is based on the principle of avalanche multiplication of the current.

* It consists of heavily doped p^+ and n^+ regions.

* The depletion region is highly doped, almost intrinsic. The diode is reverse biased using 150-300V.

* The avalanche photodiode has better noise performance, because the carrier multiplication is limited to electrons only.

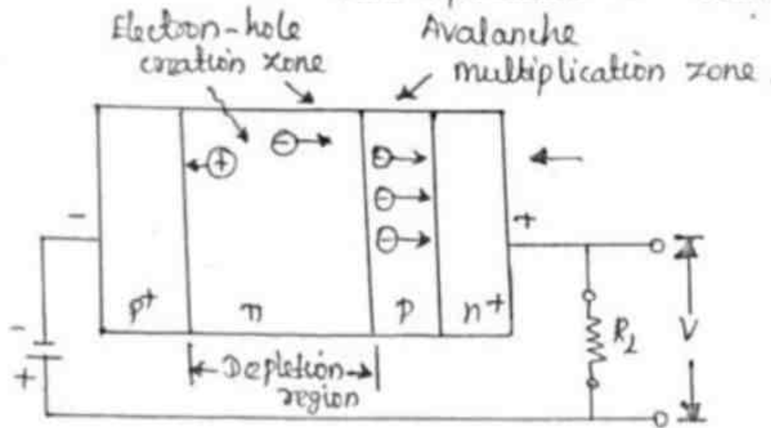
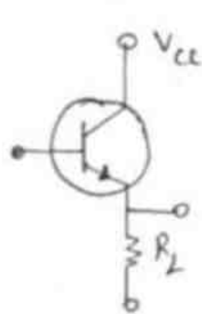
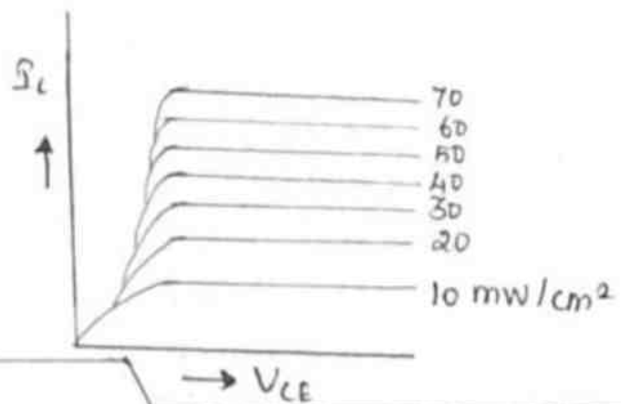


Photo - transistor :-

* Photo - Transistor is another type of photo detector.

* A transistor photo diode with its characteristic curves





Solar cell :-

* It is a P-N Junction diode which converts solar energy (light energy) into electrical energy.

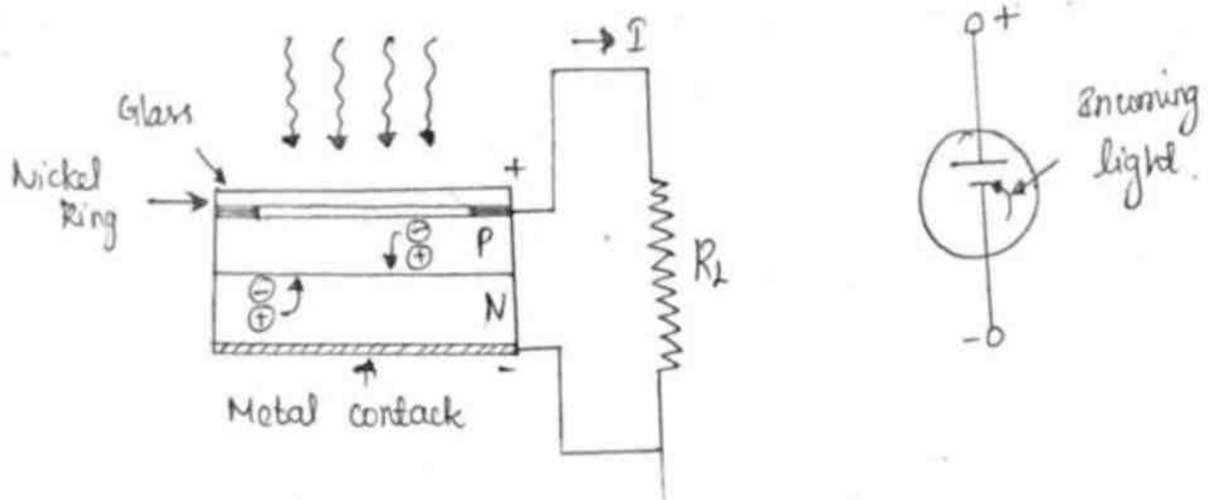
* Common materials for solar cells include silicon (Si), Gallium Arsenide (GaAs), Indium Arsenide (InAs) and cadmium Arsenide (CdAs). The most common is silicon.

* for silicon, the band gap is 1.12eV.

Construction:

* It consists of P-N Junction diode made of silicon.

The P-N diode is packed in a can with glass window on top such that light may fall upon p and N type materials.



* The thickness of the p-region is kept very small. Therefore electrons generated in p region can diffuse to the junction before recombination takes place.

* The thickness of N-region is also kept small to allow holes generated near the surface to diffuse to the junction before they recombine.

* A nickel ring is provided around the p-layer which acts as the positive output terminal.

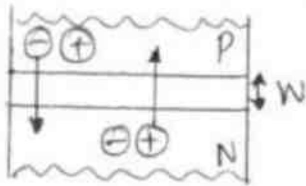
* A metal contact at the bottom serves as the negative output terminal.

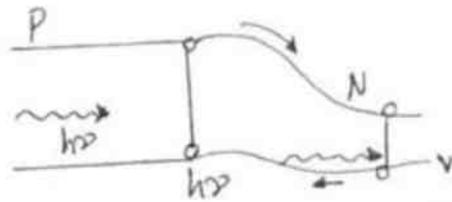
Working:-

These electrons and holes reach the depletion region by diffusion and they are separated by the strong barrier electrical field existing there.

The minority carrier electrons in the p-side cross the barrier potential to reach N-side and the holes in N-side move to the P-side

(14)

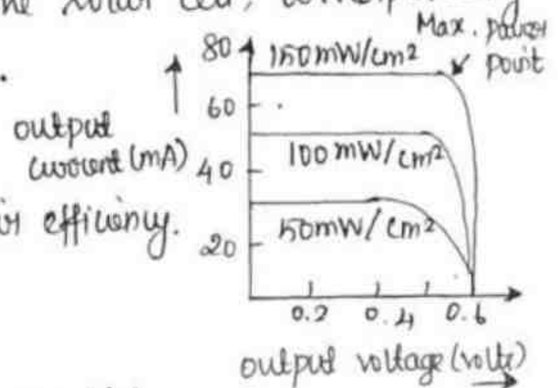




The open-circuit voltage produced for a Silicon solar cell is typically 0.6 volt and the short-circuit current is about $40mA/cm^2$ in bright noon day sun light.

V-I characteristics:-

The V-I characteristics of the solar cell, corresponding to different levels of illumination.



Advantages:-

* The solar cell operates with fair efficiency.
* It has unlimited life
* It can be mass produced
* It has a high power capacity per weight
* Its size is small and compact.

Disadvantages:-

Solar energy is not available round the clock. It cannot be obtained during night time.

Uses:-

* They are used to give power to the calculators and watches.
* They are used to provide commercial electricity.

Light Emitting Diode (LED)

It is a p-n junction diode which emits light when it is forward biased.

principle:-

The injection of electrons into the p-region from n-region makes a direct transition from the conduction band to valence band.

Then, the electrons recombine with holes and emits photons of energy Eg.

The forbidden energy gap is given by.

$$\boxed{E_g = h\nu} \quad \ldots (1)$$

where, $h \rightarrow$ plancks constant, $\nu \rightarrow$ frequency of the emitted radiation.

$$\nu = \frac{c}{\lambda} \quad \ldots (2)$$

where, $c \rightarrow$ velocity of the light, $\lambda \rightarrow$ wave length of the light.

$$\boxed{\therefore E_g = \frac{hc}{\lambda}} \quad \ldots (3)$$

Hence, the wavelength of the emitted photon is given by the relation.

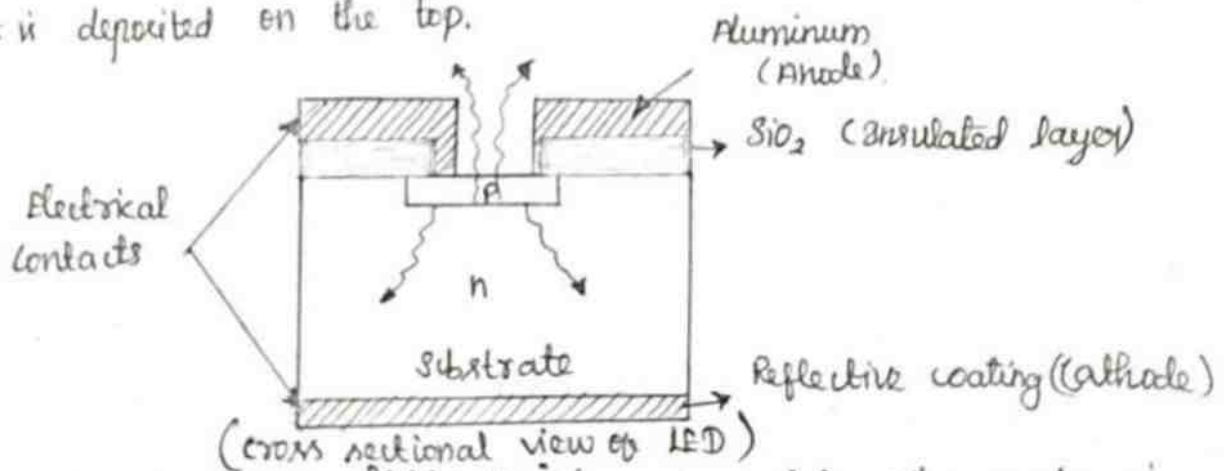
$$\boxed{\lambda = \frac{hc}{E_g}} \quad \ldots (4)$$

The wave length of the light emitted purely depends on the band gap energy.

Construction :-

A n-type layer is grown on a substrate and a p-type layer is deposited on it by diffusion.

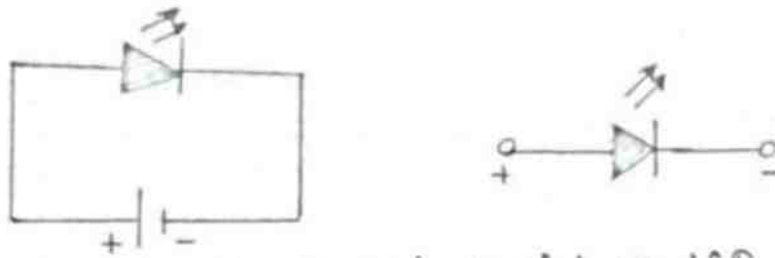
Since carrier recombination takes place in the p-layer, It is deposited on the top.



(cross sectional view of LED)

For maximum light emission, a metal film anode is deposited at the outer edges of the p-type layer.

The bottom of the substrate is coated with a metal film.



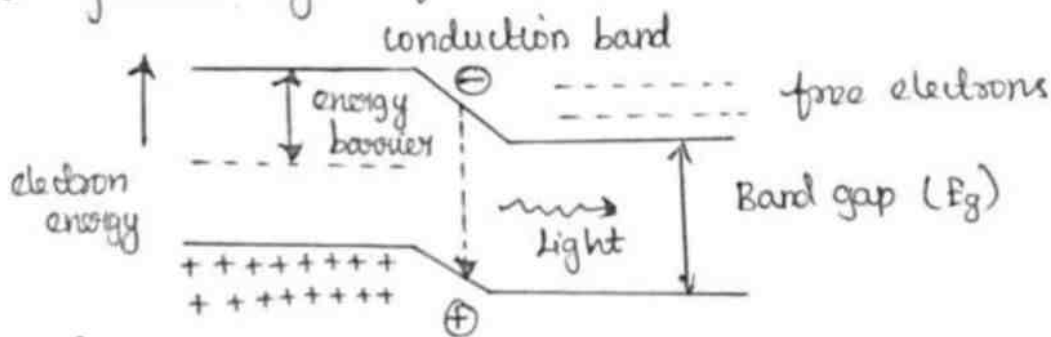
circuit and symbol of LED.

working:-

* when the p-n junction diode is forward biased, the barrier width is reduced.

* raising the potential energy on the n-side and lowering that of the p-side.

* Thus, light radiation from LED is caused by the recombination of holes and electrons that are injected into the junction by a forward bias voltage.



Advantages of LEDS

* Light modulation can be achieved with pulse supply.
* It has long life time.
* It has low drive voltage and low noise
* It is easily interfaced to digital logic circuits
* It can be operated over a wide range of temperatures.

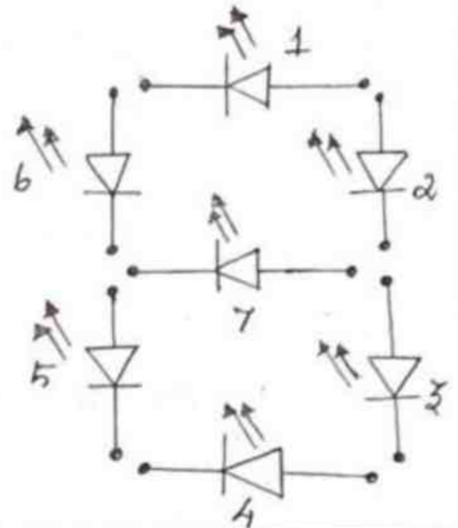
Disadvantages of LEDs:-

* They require high power.
* Their preparation cost is high when compared to LCD.

Applications and uses of LEDs.

Because of their miniature size, they are widely used in numeric and alphanumeric display devices.

* They are used as indicator lamps.
* Infrared LEDs are used in burglar alarms.
* They are used as a pilot light.
* They are used in image sensing circuits used for picture phone.



Laser Diodes:-

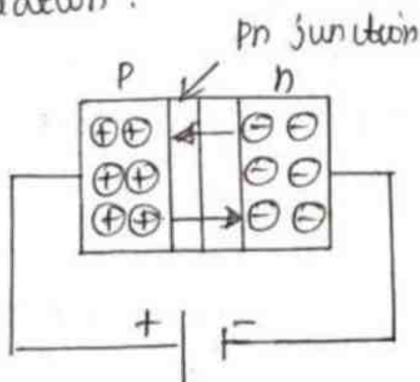
Definition:-

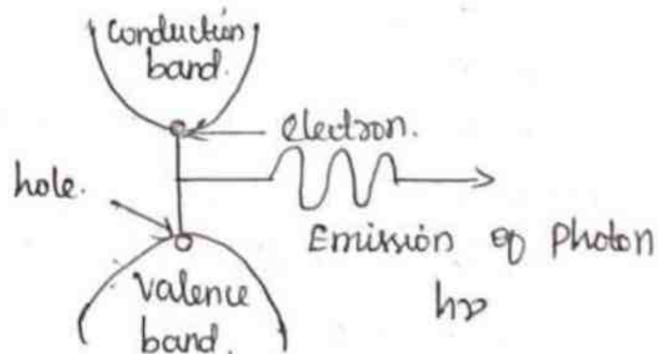
It is a specially fabricated p-n junction diode. This diode emits laser light when it is forward-biased.

principle:-

When the p-n junction diode is forward-biased, the electrons from n-region and holes from p-region cross the junction and recombine with each other.

During the recombination process, the light radiation is released from a direct band gap semiconductors like GaAs. This light radiation is known as recombination radiation.



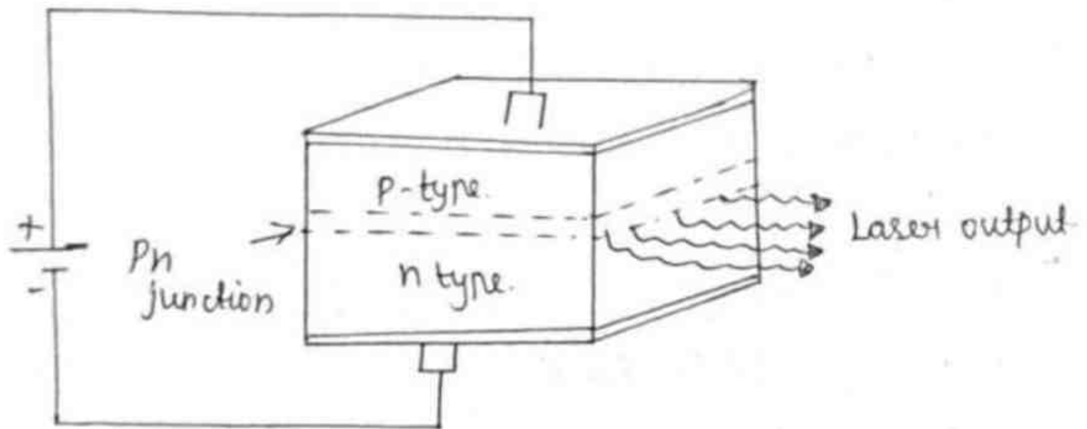


construction:-

The active medium is a p-n junction diode made from a single crystal of gallium arsenide.

This crystal is cut in the form of a platelet having a thickness of 0.5mm.

This platelet consists of two regions n-type and p-type

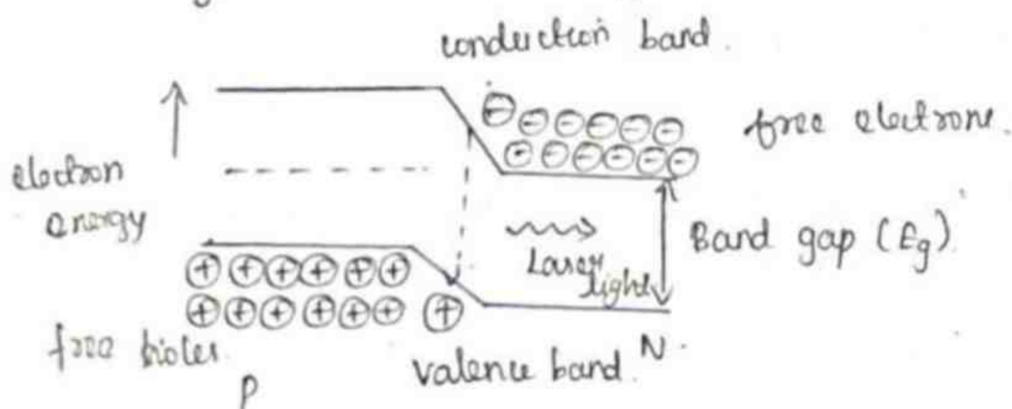


Now the photon emission is stimulated in a very thin layer of pn junction.

working :-

when the pn junction is forward-biased, the electrons and holes are injected into junction region.

Now the electrons and holes recombine with each other. During recombination, light photons are produced.



The wavelength of laser light is given by

$$E_g = h\nu = \frac{hc}{\lambda} \qquad \left(\because \nu = \frac{c}{\lambda}\right)$$

$E_g \rightarrow$ band gap in joule.

$$\boxed{\lambda = \frac{hc}{E_g}}$$

E_g

(19)

Characteristics:-

Type:- solid state semiconductor laser.

Active medium:- A pn junction diode made from a single crystal of (gallium arsenide)

pumping method:- Direct conversion method.

Power output:- continuous wave or pulsed output.

wavelength of output:- 8300 Å to 8500 Å.

Advantages:-

* This laser is very small in size and compact.
* It has high efficiency.
* It is operated with less power than ruby and CO_2 lasers.
* It requires very little additional equipment.
* It emits a continuous wave output or pulsed output.

Disadvantages:-

* Laser output beam has large divergence.
* The purity and monochromacity are poor.
* It has poor coherence and stability.

Applications of Laser Diode:-

* used in fibre optic communication.
* used in various measuring devices such as range finders bar-code readers.
* used in laser medicine especially, dentistry.

Organic semiconductors : optical processes and devices.

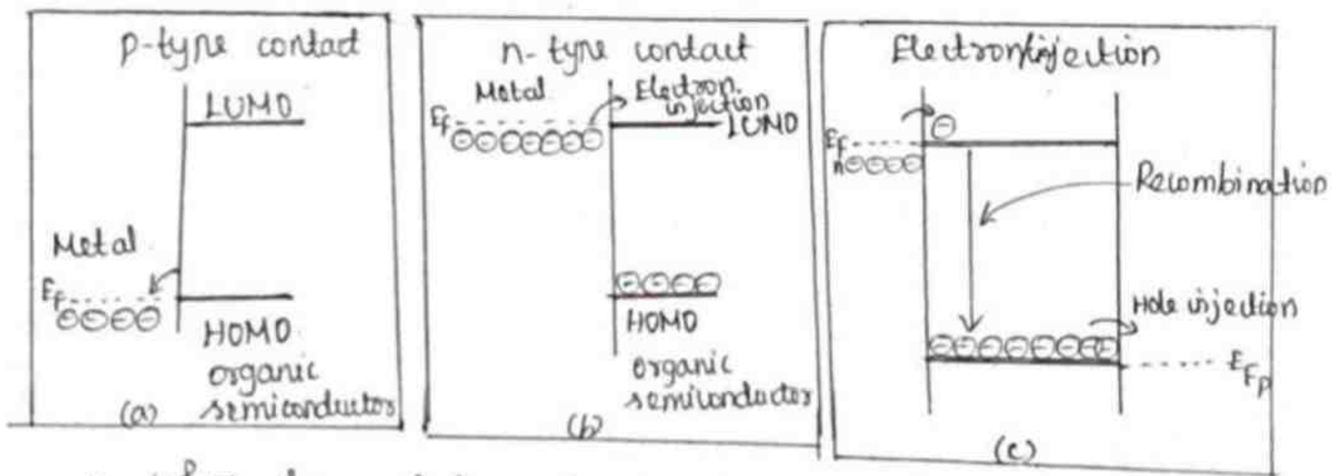
* In the last 1980s the interest in organic semiconductors demonstrated by the invention of electroluminesient devices and FETs made from polythiophene

* Polymer-based devices are now used for back-lights of liquid crystal displays, displays of devices, such as cell phones or watches.

* It is used in that commercial technologies, such as televisions, solar cells, etc,

* In organic semiconductors, the nature of atomic bonding results in very narrow range energy levels.

* When the fermi energy of the metal is close to the HOMO level.



* When the metal work functions are close to the LUMO state, an n-type contact results.

* The current flow in a p-n diode using an organic semiconductor film, is fig (c).

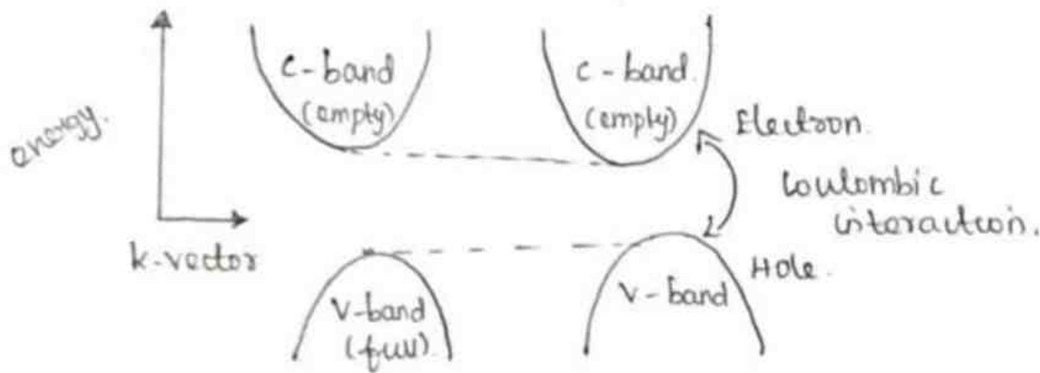
* It is a bound state of electron and hole is a e-h interacting through coulombic interaction.

EXCITONIC state:-

* The two bands are separated by the bandgap. The Organic semiconductor materials have HOMO and LUMO bands the like valence band conduction bands in inorganic semiconductor

* This gives a "single - electron energy - momentum" relation

* The electron and hole interact with other, The bound state of electron-hole system is called exciton and it can be represented by the hydrogen - like model.



* The exciton creates photons. these photons are quite different in energy from the LUMA - HOMO energy difference.

* The process of light emission involves.

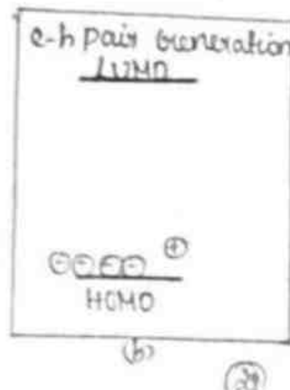
(i) injection of electrons from the contacts,

(ii) diffusion of the carrier in the LUMO or HOMO states,

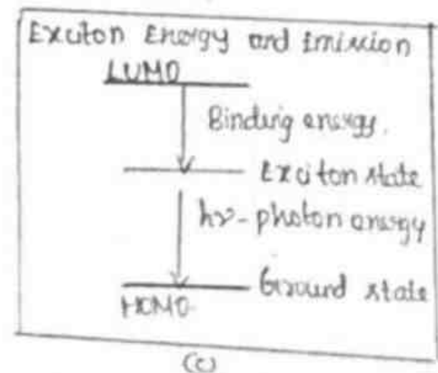
iii) exciton formation,

iv) exciton recombination to emit a photon.

(a)



(b)



(c)

Unit-5

Nano Devices.

1. Density of states for solids:-

Definition:

Electron density is the number of electrons per unit volume in a material. It is determined by using density of states.

Density of states:-

In solid, the total number of electron energy states N with energies upto E is determined based on quantum mechanics using the following equation.

$$N = \left(\frac{8\pi}{3}\right)(2mE)^{3/2}\left(\frac{a^3}{h^3}\right) \quad \ldots (1)$$

$a^3 \rightarrow$ the volume of the material.

$E \rightarrow$ maximum energy level

$m \rightarrow$ mass of an electron

$h \rightarrow$ planck's constant.

Number of energy states per unit volume.

$$n = \frac{N}{a^3} = \left(\frac{8\pi}{3}\right)\frac{(2mE)^{3/2}}{h^3} \quad \ldots (2)$$

$$Z(E) = \frac{dn}{dE}$$

$$= \frac{d}{dE}\left(\frac{8\pi}{3}\frac{(2mE)^{3/2}}{h^3}\right)$$

$$= \frac{8\pi}{3}\frac{(2m)^{3/2}}{h^3}\frac{d(E)^{3/2}}{dE}$$

$$= \frac{8\pi}{3}\frac{2^{3/2}m^{3/2}}{h^3}\left(\frac{3}{2}E^{3/2-1}\right)$$

$$= \frac{8\pi}{3} \frac{2^1 \times 2^{1/2} m^{3/2}}{h^3} \times \left(\frac{3}{2} E^{1/2} \right)$$

$$\boxed{Z(E) = \frac{8\pi \sqrt{2}\, m^{3/2}}{h^3} \sqrt{E}} \quad \ldots (3).$$

The number of free electrons per unit volume, or electron density, in a bulk conductor at 0K using eqn (2).

$$\boxed{n_e = \frac{8\pi}{3} \left(\frac{(2m E_F)^{3/2}}{h^3} \right)} \quad \ldots (4). \quad (\because E = E_F)$$

The minute size of the nanomaterials gives unique electronic properties.

2. Significance between fermi energy and volume of the material.

Fermi energy :-

It is defined as the highest energy level occupied by the electron at 0k in metal.

Now rearranging eqn (4) for the fermi energy of a conductor,

$$E_F = \frac{h^2}{2m} \left(\frac{3 n_e}{8\pi} \right)^{2/3} \quad \ldots (5).$$

Here, n_e is the only variable and all the other terms are constants.

$$\boxed{E_F \propto n_e^{2/3}} \quad \ldots (6).$$

E_F is the same for a particle of a solid as it is for a solid itself.

The average spacing between energy states is inversely proportional to the volume of the solid.

$$\Delta E \propto \frac{1}{a^3} \quad \ldots \quad (7)$$

3. Quantum confinement :-

Definition:-

* It is a process of reduction of the size of the solid such that the energy levels inside become discrete.
* In this case, small "droplets" of isolated electrons are created. Thus, the energy of a small volume of such materials are quantized just like in an atom.
* This type of artificial or fake atoms have tunable electrical properties.
* The quantum confinement effect is observed when the size of the particle is too small to be comparable to the wavelength of the electron.
* In bottom-up approach, low-volume structures are built atom by atom.
* In the top-down approach, material is removed from one or more of three dimensions of larger solid.
* In both cases, a structure small enough for quantum behavior to manifest can be produced.

4. Quantum structures:-

Definition:-

When a bulk material is reduced in its size, atleast one of its dimension, in the order of few

nanometres, then the structure is known as quantum structure.

Explanation :-

A structure in which the motion of the electrons or holes are confined in one or more directions by potential barriers is called quantum confined structure.

The quantum confined structure is classified into three types based on the confinement directions. They are,

* quantum well
* quantum wire
* quantum dot

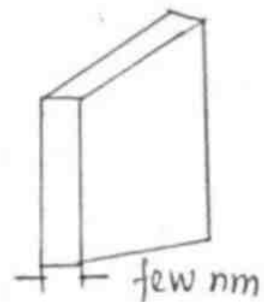
(i) quantum well (2- dimension)

Definition:

when the electrons are confined inside a region of minimal width. confinement in one dimension "quantum-well" is created.

construction:

Quantum wells are made from alternative layers of different semiconductors or by deposition of very thin metal films.



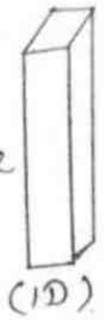
Explanation :-

* The quantum well is a larger structure in which the carrier particles are free to move in two dimensions.
* The particles are confined in one dimension hence, they are considered to be "quantum confinement."

use :-

Quantum wells are now widely used to make semiconductor lasers and those others important devices.

ii) Quantum wire (1 dimension)



Definition:-

When the electrons are confined in two mutually perpendicular directions, then the structure is known as quantum wire.

Explanation:-

* The carriers trapped in such structure are considered to be 1-D quantum confinement.

* In this case, a carrier is only free to move its trajectory along the wire.

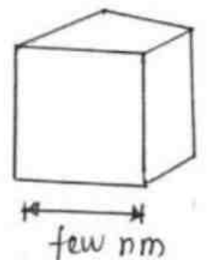
Example:-

Example of quantum wire structure are nanowires, nanorod and nanotube.

iii) Quantum Dots (0-Dimension)

Definition:-

when all the three dimensions are minimized the resulting structure is known as quantum dot.



Explanation:-

* In quantum dot, the carriers has only confined states. i.e, there are no freely moving carriers.

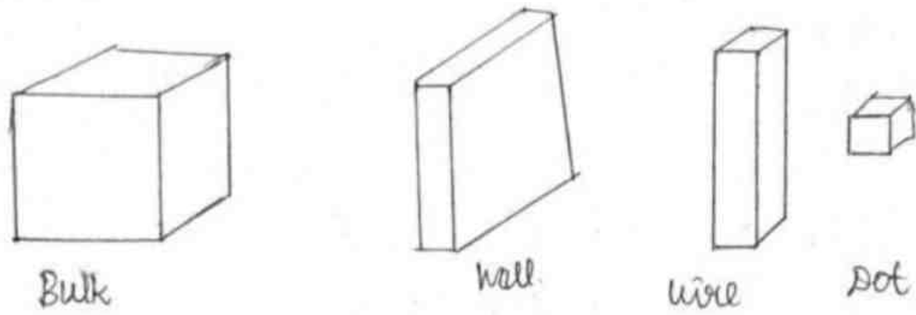
* Though a quantum dot has many thousands of atoms carriers it is considered more like a single atom due to its peculiar properties.

Use:-

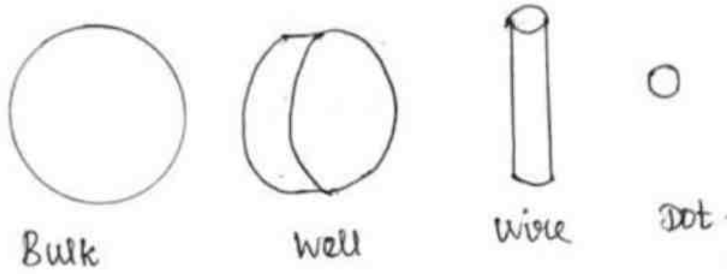
Quantum dot can be used in a quatum computer and lasers etc.

The processes of diminishing the size in the case of rectilinear geometry and curvilinear geometry,

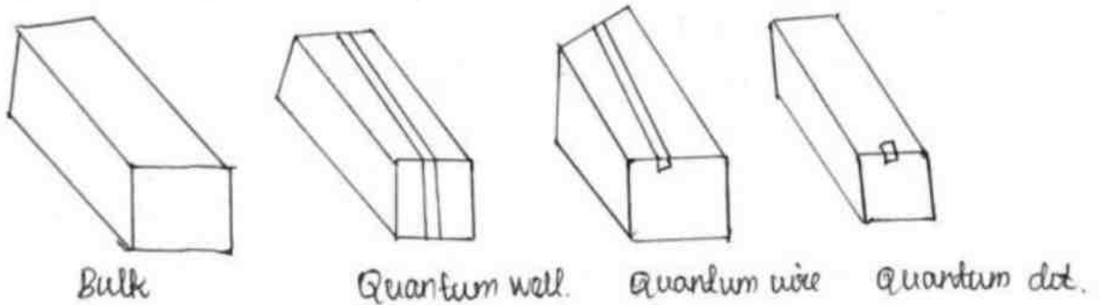
(F)



Progressive generation of rectangular nanostructures.



Progressive generation of curvilinear nanostructures.



Three quantum structure.

The classification of quantum confined structures.

Structure	Quantum confinement directions	Number of unconfined dimensions
Bulk	0	3
Quantum well	1	2
Quantum wire	2	1
Quantum dot	3	0

5. Density of states in Quantum well, Quantum wire and Quantum Dot structure.

Bulk structure :- (fig (a))

The density of state of a bulk material is given by,

$$Z(E) = \frac{8\pi\sqrt{2}\, m^{*\,3/2} (E-E_c)^{1/2}}{h^3}$$

where, E_c - bottom of conduction band energy.

m^* - effective mass of electron.

Quantum well structure :- (fig (b))

In quantum well structure, the electrons can move freely in two directions and confined in only one direction.

The density of state of the quantum well structure.

$$Z(E) = \frac{4\pi m^*}{h^2} \qquad E_0 > E_i \qquad i = 1, 2, 3$$

Quantum wire structure :- (fig (c))

* The quantum wire structure provides only one non confinement direction.
* The carrier can move freely along one direction
* The remaining two directions are confined for charge carrier.

The density of states of quantum wire is proportional to $\frac{1}{\sqrt{E}}$. The density of state for this structure is given as.

$$Z(E) = \frac{2\sqrt{2}\, m^* (E-E_i)^{-1/2}}{h} \qquad i = 1, 2, 3$$

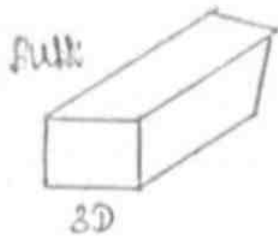
Quantum dot:- (fig (d))

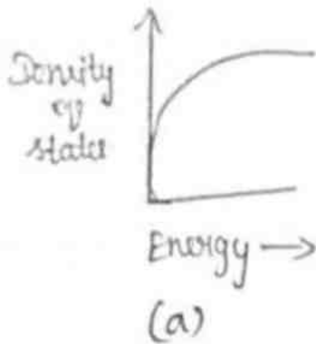
* In quantum dot all the directions are confined and no direction in which electron movement is free.

* The density of state for this structure is given by,

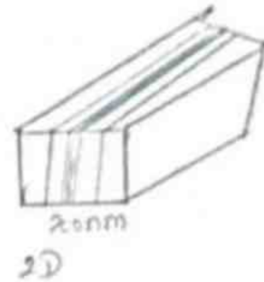
$$Z(E) = \delta(E - E_i), \; i = 1, 2, 3$$

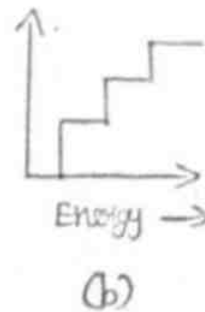
For quantum dot (0D) or nano particle, the permitted energy values are not continuous but form discrete bunches of varying densities.



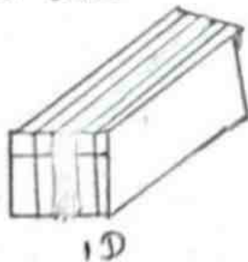


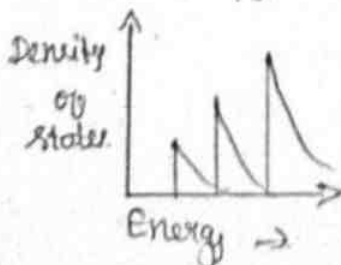
(a)

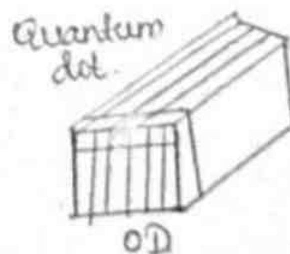


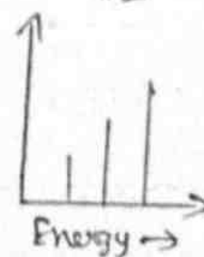


(b)







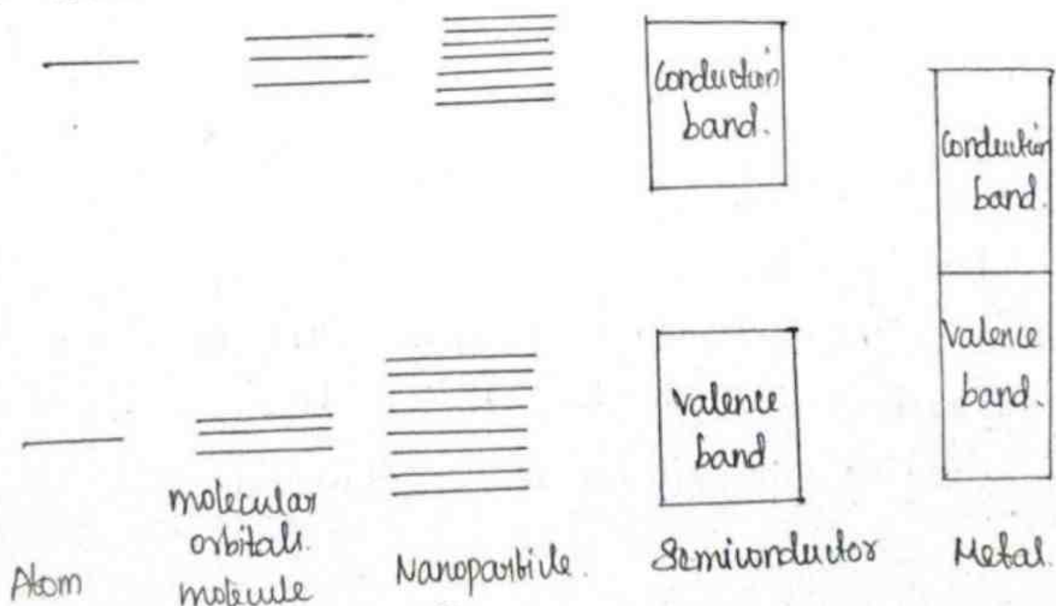


Density of states as a function of dimensionality of the Structure.

Types of Structure & No. of non-confinement Dimensions	Density of state fn.
Bulk (3D)	$Z(E) = \frac{8\pi\sqrt{2}\, m^{*3/2} (E-E_c)^{1/2}}{h^3}$
Quantum well (2D)	$Z(E) = \frac{4\pi m^*}{h^2}$ $E > E_i$
Quantum wire (1D)	$Z(E) = \frac{2\sqrt{2}\, m^* (E-E_i)^{-1/2}}{h}$
Quantum dot (0D)	$Z(E) = \delta(E-E_i)$.

6. BandGap of Nanomaterials:-

* The electronic properties of metals and semiconductors are determined by their electronic band structure.
* The band structure changes with particle size.
* Molecular orbitals get converted into delocalized band states.



The particles that show this size quantization effect are called Q - particles or quantum dots.

In metals, the quantum size effect exists but it can be seen only in particles smaller than 2nm where localization of energy levels can be observed when the spacing between the levels exceeds thermal energy.

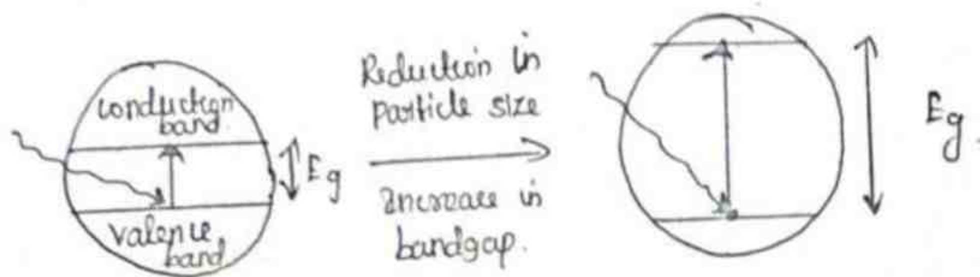
Quantum size effect:-

* When the size of a nanocrystal becomes smaller than the de Broglie wavelength, electrons and holes get spatially confined, electrical dipoles get generated, the discrete energy levels are formed.

* As the size of the particle decreases, absorption shifts towards the shorter wavelength.

* This indicates an increase in the bandgap energy.

* A change in absorption causes a change in the colour of the semiconductor nanoparticle.



Tunneling:-

The phenomenon of penetration of charge carriers directly through the potential barrier, instead of climbing over it, is called tunneling.

conductivity of metallic Nanowires:-

Let us study what happens when wire length L becomes extremely small relative to the mean free path and the influence of wire cross-section size on conductivity of metallic nanowires.

Consider a circular cross-section wire which has radius a and length L. Assume that L is very large compared to the mean free path.

A relativity simple approximate formula for the resistivity (ρ) of rectangular cross-section wires.

$$\rho = \rho_0 \left\{ \frac{1}{3\left[\frac{1}{3} - \frac{\alpha}{2} + \alpha^2 - \alpha^3 \ln\left(1+\frac{1}{\alpha}\right)\right]} + \frac{3}{8} C (1-p) \frac{1+AR}{AR} \frac{L_m}{w} \right\} \quad \cdots (1)$$

where,

$$\alpha = \frac{L_m}{d} \frac{R_c}{(1-R_c)}$$

ρ_0 - bulk resistivity.

w - wire width.

AR - aspect ratio

d - average grain size.

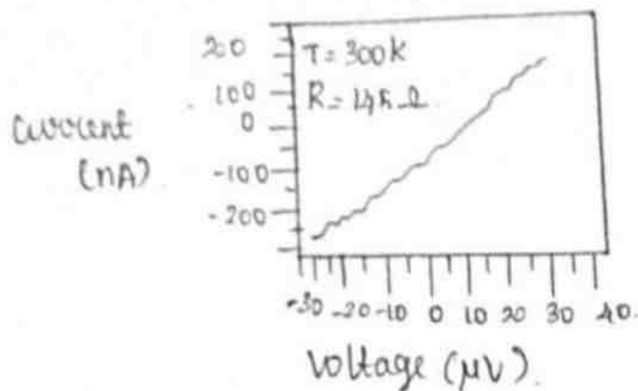
p - specularity parameter.

R_c - grain boundary reflectivity coefficient.

C - a constant.

However, as complicated as surface and grain-boundary scattering are other factors also determine the conductivity of a nanowire.

for example, the V-I characteristic of a 30 nm radius, 2.4 μm long single-crystalline copper nanowire



Ballistic Transport:-

The conductivity is a bulk parameter. It is derived by assuming a large number of electrons and a large number of collisions between electrons and phonons impurities, imperfections, etc.

L → conduction path is reduced to become much less than the mean free path Lm.

Definition:-

When the dimension of the conductor (L) is smaller than the mean free path of the electron (Lm) then the transport of electron is called ballistic transport.

Explanation:-

when the length 'L' of the conductor becomes much smaller than the mean free path 'Lm' the transport is termed "ballistic".

It means that the electrons do not scatter during the time, it travels through the conductor.

Example:- ballistic transport can be observed in a metal nano wire.

Condition of Ballistic transport:-

The mean free path can be increased by reducing the number of impurities in the crystal or by lowering its temperature.

Ballistic transport conditions are,

i) $L \ll L_m$ and ii) $L \ll L_\phi$

$L \rightarrow$ Length of the conductor, $L_m \rightarrow$ Mean free path.
$L_\phi \rightarrow$ Length over which an electron can travel before having an inelastic collision. This is also called the phase-coherence length.

Application:-

* One practical application of ballistic transport is to ultra-short-channel semiconducting FETS or carbon nano tube transistors.

* That is in a very small length scales, electrons transport occurs ballistically.

Quantum Resistance and conductance:-

* If the wire is short enough, ie, shorter than the electron mean free path in the material, there will be no scattering and the transport is ballistic.

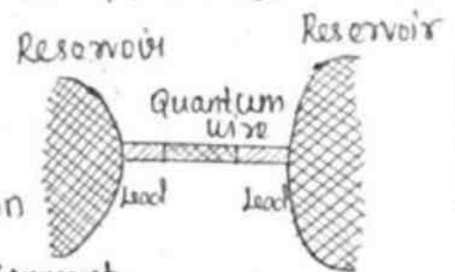
* As in figure, the 1D quantum wire is connected through ideal leads.

* They do not produce scattering events to reservoirs characterized by fermi levels E_{F1} and E_{F2}.

* In order to flow of the current through the quantum wire, a small voltage V is applied b/w the

reservoirs.

* As a consequence, there is a potential energy eV between the two reservoirs equal to $E_{F1} - E_{F2}$.



$$I = e n_{1D}(E) v(E) eV \quad \cdots (1)$$

Substituting for $n_{1D}(E)$ by its expression we obtain the following value of the current.

$$I = \frac{2e^2}{h} V \quad \cdots\cdots (2)$$

It is independent of the carrier velocity.

The value of the conductance $G = (I/V)$,

$$G = \frac{\frac{2e^2 V}{h}}{V} \Rightarrow G = \frac{2e^2}{h} \quad \cdots (3)$$

The quotient, $$G_0 = \frac{e^2}{h} \quad \cdots (4)$$

is called the quantum unit of conductance. The quantum resistance value is given by.

$$R_0 = \frac{h}{e^2} = 25.81\, k\Omega \quad \cdots (5)$$

This can be experimentally determined.

Since the quantity $\frac{2e^2}{h}$ appears very often, it is usually called fundamental conductance.

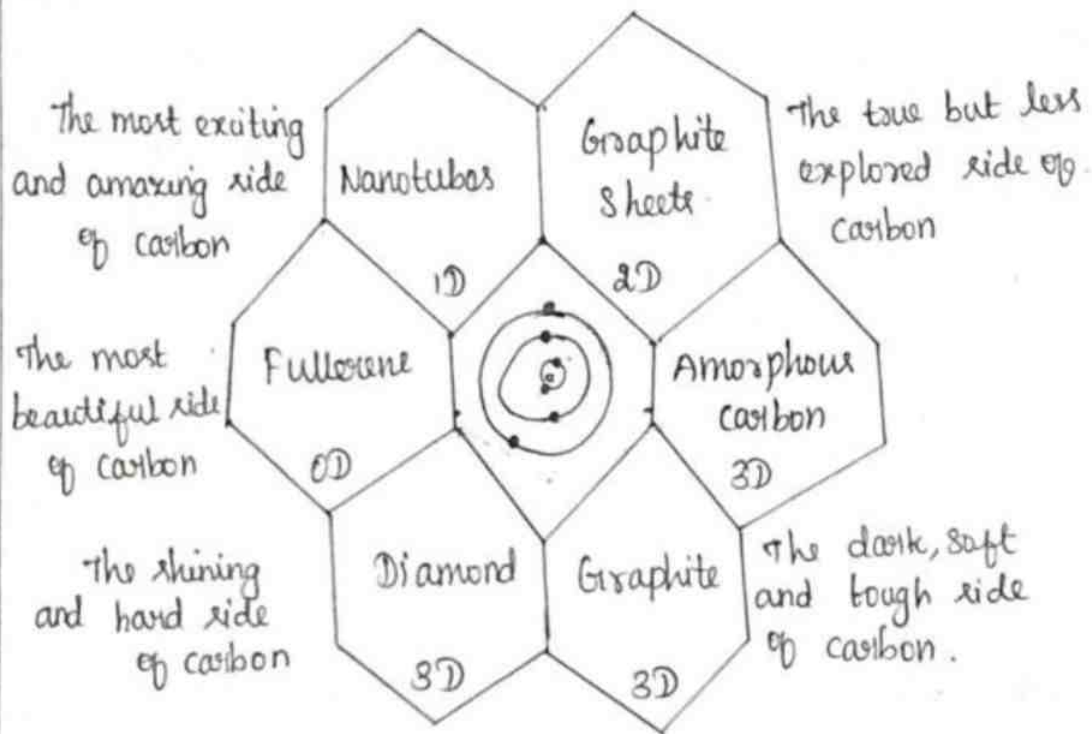
Carbon:-

In nanotechnology, researchers are looking out for certain materials with desired properties through.

which the nanoscale components and structures can be obtained.

* Carbon is found to be one such material suitable for nanotechnology based components, due to its inherent desirable properties.

* Pure carbon exists in four different crystalline forms namely Diamond, Graphite, fullerenes and Nanotubes.

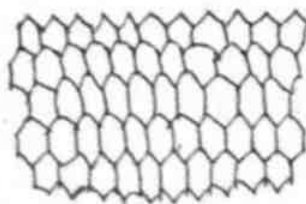


Carbon Nanotubes (CNT)

* A group of nanostructures with large potential applications are carbon nanotubes.

* The hexagonal lattice of carbon is simply graphite.

* A single layer of graphite is called graphene

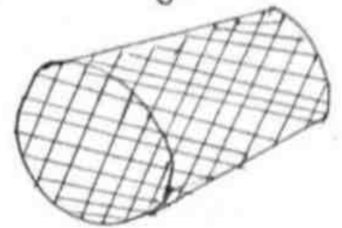


Graphene sheet

* When the graphene layer is rolled, the structure is tube like and it is a single molecule.

* The layer appears like a rolled-up chicked wire with carbon atoms at the apexes of the hexagon.

* The carbon nanotubes are hollow cylinders of extremely thin diameter, 10,000 times smaller than a human hair.



SW CNT
graphene sheet

Structure of CNT :-

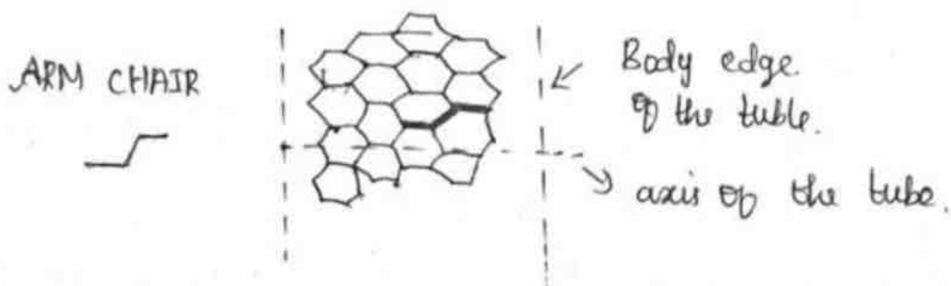
* The CNTs have many structures on the basis of their length type of spiral and number of layers.

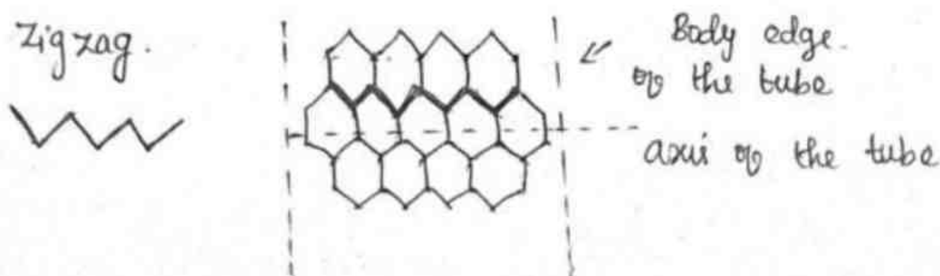
* Their electrical properties depend on their structure and they act as both a metal or a semiconductor.

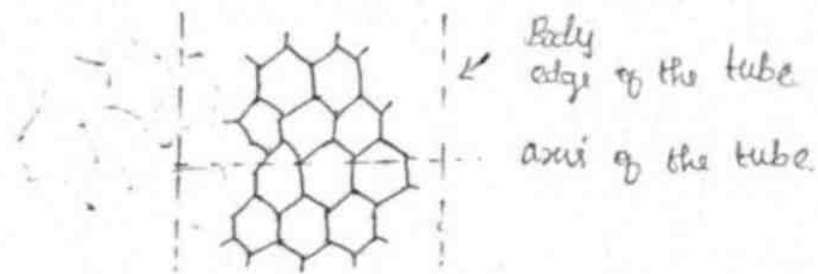
Types of CNT Structures :-

Three types of nanotube structures are considered by rolling a graphite sheet with different orientations about the axis.

(i) Armchair structure, (ii) zig-zag structure, (iii) chairal structure







Armchair structure:-

when the axis of the tube parallel to c-c bonds of the carbon hexagons the structure. It is referred as "armchair" structure.

Zig-zag and chairal structure:-

* The tubes sketched in figure, referred as zig-zag and chiral structure.

* They are formed by rolling graphene sheet such that the axis of the tube is not parallel to c-c bonds.

* Zig-zag structure consists of tube axis perpendicular to c-c bonds.

* In chairal structure, c-c bond is inclined towards the axis of the tube.

classification of CNT:-

Based on the number of layers, the carbon nanotubes are classified as,

* Single-walled (SWNTs)
* Multi-Walled (MWNTs)

In multi-walled nanotubes more than one CNTs are coaxially arranged.

Properties of CNTs

a) Electrical properties:-

(i) Carbon nanotubes are metallic or semiconducting depending on the diameter and chirality.

ii) The energy gap of semiconducting chiral carbon nanotubes is inversely proportional to the diameter of the tube.

iii) The energy bandgap decreases with increase of diameter of the CNTs.

This means that nanotubes can be treated as quantum wires atleast at very low temperature.

b) Mechanical properties:-

* The strength of the carbon - carbon bond is very high therefore any structure based on aligned carbon - carbon bonds will ultimately have high strength.

* Youngs modulus of CNT is about 1.8 TPa, Nanotubes have therefore high ultimate tensile - strength.

* One of the important properties of nanotubes is their ability to withstand extreme strain.

c) Physical properties:-

* Nanotubes have a high strength - to - weight ratio.

* This is indeed useful for lightweight applications.

* This value is about 100 times that of steel and over twice that of conventional carbon fibres.

* The surface area of nanotubes is of the order of 10-20 m^2/g which is higher than that of graphite.

(18)